DEPARTMENT OF THE ARMY
US Army Corps of Engineers
Washington, DC 20314-1000

CECW-ED

Engineer Technical
Letter No. 1110-8-17(FR)

15 April 1992

Engineering and Design
PILE LAYOUT TO MINIMIZE INTERFERENCE

1. Purpose

This engineer technical letter (ETL) provides guidance for designing pile foundation layouts that will minimize the risk of pile interference.

2. Applicability

This ETL applies to all US Army Corps of Engineers (USACE) elements and USACE Commands having civil works responsibilities for the design of civil works projects.

3. References

a. EM 1110-2-2906. "Design of Pile Foundations."

b. Civil Works Construction Guide Specification CW-02315. Apr 1990. "Steel H-Piles."

c. Wolff, Thomas F. Apr 1990. "PILINT: A Software Package Designed for Pile Interference Analysis," Research report prepared for US Army Corps of Engineers, Michigan State University.

d. Wolff, Thomas F. Sep 1990. "User's Guide: Pile Group Interference Probabilistic Assessment (CPGP) Computer Program," Contract report to US Army Engineer Waterways Experiment Station.

4. Summary

a. Deterministic method. In many cases, adherence to normal specification tolerances for pile location and alignment (Reference 3b) will eliminate the possibility of pile intersection. The deterministic solution for the minimum pile spacing required to preclude intersection is provided in Enclosure 1.

b. Probabilistic method. Where long piles are to be driven at close spacings, there may be some likelihood of pile intersection. Recent research (References 3c and 3d) has developed a probabilistic method and computer solution (CPGP) to assess the risks of such intersections. The work is summarized in Enclosure 1. The probability of intersection for a single interior pile in a large group is determined as a function of pile diameter, length, spacing, camber, batter, and the expected standard deviations of the ground placement errors and alignment errors. The probability distribution for the number of intersections in the group is determine from the probability of intersection for a single pile and the layout of a group.

c. Chart solutions and examples. Pile layout to minimize interference is provided in Enclosure 2. Chart solutions for the probability of intersection of an interior vertical pile in a uniformly spaced group have been developed using CPGP and are provided in Enclosure 3. For batter piles and other cases not covered by the chart solutions, the probability of intersection for an interior pile can be determined using CPGP. In both cases, the expected number of intersections and the probability distribution for the number of intersections in an entire group can be determined using CPGP. Three example problems are provided in Enclosures 4 through 6.

d. Risks and consequences. Where accurate cost information can be assigned to the occurrence of a pile intersection, the economic consequences can be determined as the product of the expected number of intersections times the cost per intersection. The cost of intersection may include costs of pulling damaged piles, furnishing and redriving new piles, and delay costs. This idea should not be construed as a directive to design for the minimum costs allowing for intersection. When making design decisions, such an analysis can provide a quantitative means to assess the risk associated with designing a pile layout where intersection may theoretically occur but is statistically unlikely. The example in Enclosure 4 illustrates such an analysis.

5. Action

a. Pile layouts. In normal circumstances, pile layouts and specification tolerances (Reference 3a) should be developed so as to eliminate the possibility of pile intersection. The deterministic method described in Enclosure 1 should be used to determine the minimum pile spacing necessary to prevent intersection. For foundations requiring long piles or close pile spacings, it may not be practical to prescribe layouts and tolerances that preclude theoretical intersection. For these cases a probabilistic analysis should be made using CPGP as described in Enclosure 1. The combination of pile diameter, length, and spacing for final designs should be such that the probability of intersection for individual piles is less than 0.002 and the expected number of intersections for groups is less than 0.5. Requests for deviations from these criteria should be made in consultation with the Office, Chief of Engineers (OCE) and should include an assessment of risks and consequences.

b. Specifications. Pile driving specifications should prescribe a course of action for suspected cases of pile intersection. This should include a requirement for measurement of the as-driven location and alignment to verify that specified tolerances were met before ordering a pile pulled for inspection. Enclosure 2 provides suggested specification wording.

c. Actual pile placement data. In the probabilistic method, the probability of intersection for single piles is a function of the standard deviation of the pile placement and alignment errors. The recommended default values for these standard deviations were obtained from data on one major project but are believed to be consistent with general practice. Districts are encouraged to expand this data base by measuring as-driven locations and alignment of driven piles.

FOR THE DIRECTOR:

6 Encl

JOHN A. McPHERSON
Acting Chief, Engineering Division
Directorate of Civil Works

PILE LAYOUT TO MINIMIZE INTERFERENCE
SUMMARY OF DETERMINISTIC AND PROBABILISTIC METHODS

1. Construction Specifications and Tolerances

The theoretical ground location for piles is normally shown on construction drawings and laid out in the field by a survey crew. As it is impractical to drive a pile at precisely the theoretical location, specifications provide for some tolerance or deviation between the theoretical location and actual location. A commonly specified tolerance (References a, b, e, f, and h) is 3 in., although deviations as great as 6 in. are sometimes allowed for timber piles, where the nonuniform shape makes it difficult to hold the pile in position during driving. Likewise, it is impractical to drive a pile precisely vertical or precisely at the specified angle or batter. Specifications usually allow a deviation from vertical or from the theoretical batter in the range of 0.15 to 0.50 in./ft, with 0.25 in./ft (Reference b) being a common value.

2. Spacing to Avoid Interference -- Deterministic Solution

Assume that two adjacent piles of length L are driven at the maximum permissible deviation, Δx_{max}, from the specified location and are inclined at the maximum permissible deviation from plumb, Δp_{max}. If these deviations are combined in the most unfavorable directions, as shown in Figure 1-1, a minimum pile spacing can be determined that will ensure that no intersections occur, providing that the piles are in fact driven within the tolerances. Neglecting the minor difference in apparent pile diameter due to the pile inclination, the resulting minimum center-to-center pile spacing, AX_{min}, is:

$$AX_{min} = 2 \text{ piles} * 1 \text{ ft}/12 \text{ in.} * [\Delta x_{max} + (L\Delta p_{max}) + D/2]$$

or

$$AX_{min} = (1/6)(\Delta x_{max} + L\Delta p_{max} + D/2)$$

where

AX_{min} is the minimum allowable center-to-center pile spacing in feet

Δx_{max} is the maximum permissible ground location error in inches (typically 3 inches)

Δp_{max} is the maximum permissible inclination error in inches per foot (typically 0.25 in./ft)

L is the pile length in feet

D is the pile diameter or width in inches

Where AX_{min} is less than the specified pile spacing, pile intersection can only occur if the piles are driven out of tolerance, and no further studies are necessary.

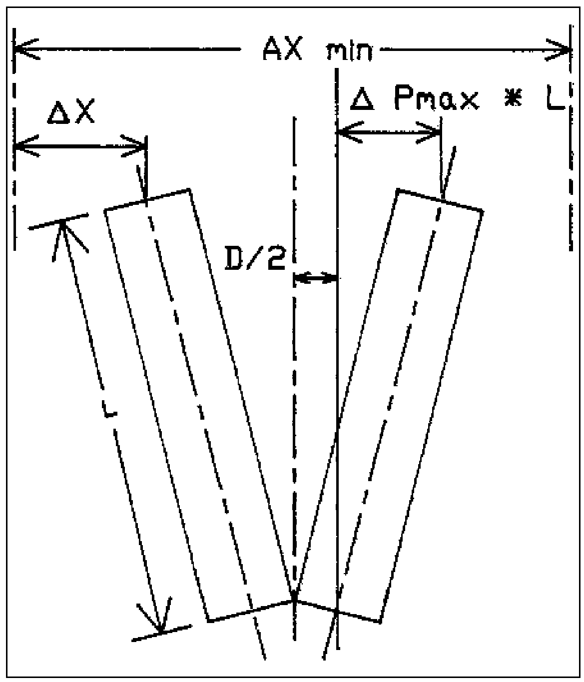


Figure 1-1. Deterministic solution for minimum pile spacing

3. Spacing to Avoid Intersection -- Probabilistic Solution

For a pile layout consisting of 14 in. diameter piles 100 ft long, the equation above gives a minimum spacing of 70 in. or 5.833 ft; hence piles spaced on 5 ft centers could theoretically intersect. However, the probability of one or more such intersections may be quite low, and perhaps tolerable. As shown in Figure 1-2, the location of a pile at any depth

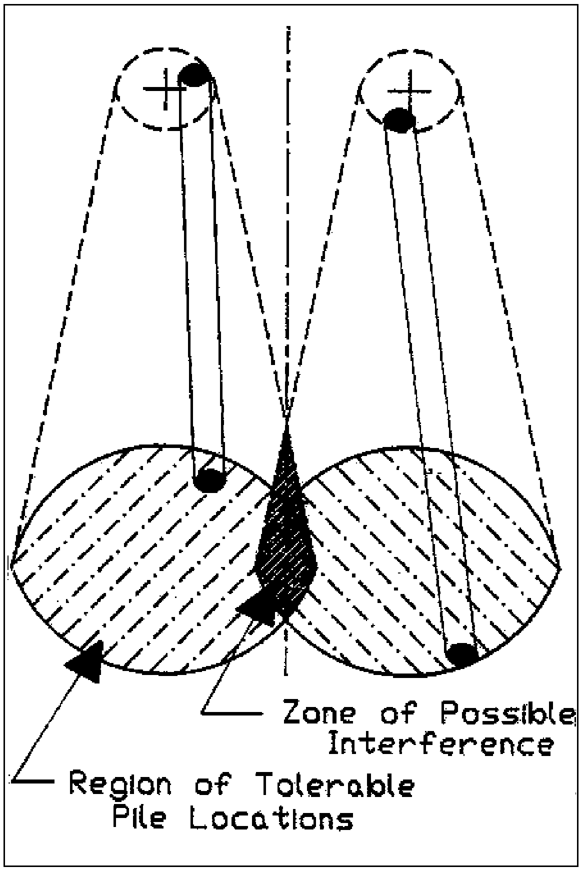


Figure 1-2. Basis of probabilistic solution

may be viewed as a two-dimensional area at a random location in a much larger area representing all the possible locations. Furthermore, certain locations, such as those corresponding to small deviations, are more probable than other locations, such as those corresponding to extreme deviations. For an intersection to occur, the pile location at some depth must overlap that of another pile, which is also random. The coincidence of two random locations overlapping at some depth has a probability which can be calculated, at least approximately. The resulting probability value can then be used to assess the probability of one or more intersections occurring in a group of a given size. A method and computer solution for assessing such probabilities have been developed (Reference j). The basis of the method is summarized below. A detailed user's guide for the program package, CPGP, is available (Reference f).

4. Estimating the Probability of Intersection for a Single Interior Pile

a. Assumptions. A typical interior pile in a large group is illustrated in Figure 1-3. The piles are assumed to be uniformly spaced at distances AX in the x direction and AY in the y direction. Furthermore, the piles are assumed to be round with diameter D and length L. As the intent of the analysis is to estimate the order of magnitude of the intersection probability rather than a precise value, rectangular piles and other shapes are modeled as equivalent round piles. The piles are assumed to be rigid; hence intersections are assumed to occur only from unfavorable location and alignment combinations, not from deflection by a boulder or similar obstruction.

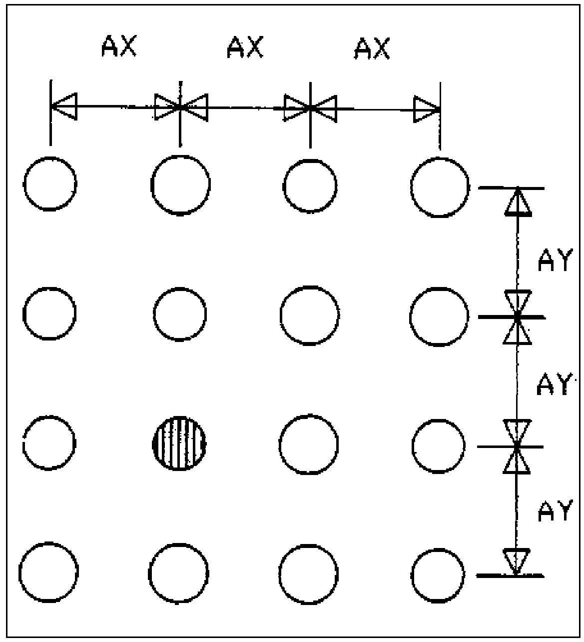


Figure 1-3. Typical interior pile in group

b. Definition of variables. The relationship of the theoretical pile location to the driven location and alignment are shown in Figure 1-4. The ground location of a driven pile is assumed to deviate from the theoretical location by two placement error components, Δx and Δy. The slope of the pile is assumed to deviate from the vertical or the theoretical batter by two alignment error components, the batter error, Δb, and the plumb error, Δp, where Δb and Δp are expressed in inches/foot. The batter error is defined in the plane of the crane boom travel, and the plumb error is defined perpendicular to the batter error. In the field, the batter error may

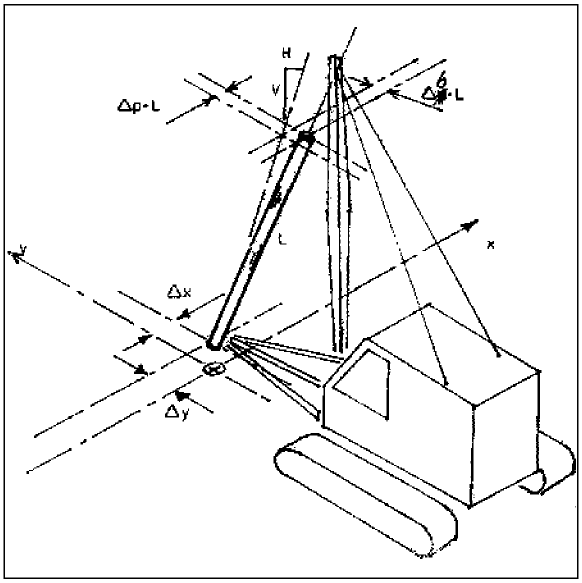


Figure 1-4. Definition of placement and alignment errors

be smaller than the plumb error as the crane operator has better control when aligning the pile and leads in this plane. The four error variables, Δx, Δy, Δb, and Δp are *random variables*. They cannot be assigned specific values as their values vary from pile to pile, but they can be defined in terms of a probability distribution. That is, probabilities can be associated with the value of the random variable being greater than or less than any particular value.

c. Probabilistic analysis. If one assumes that reasonable probability distributions can be defined for the four error variables, the probability that the axis of the pile will pass through any point in the ground can be determined. In concept, the probability that the pile will intersect another is determined by calculating the probability that the first pile will pass through a given point and a second pile will pass through the same point, and then integrating or summing over all such possible points and all possible second piles. The software package CPGP solves for the probability of intersection using a random number simulation or *Monte Carlo* analysis. Rather than perform complex integrations, the program repeatedly simulates the driving of a pile and eight surrounding piles, as shown in Figure 1-5. For each trial simulation, random values for the four error variables are generated for each of the nine piles, for a total of 36 random values. The axis locations of the piles are calculated, and a check is made whether the distance

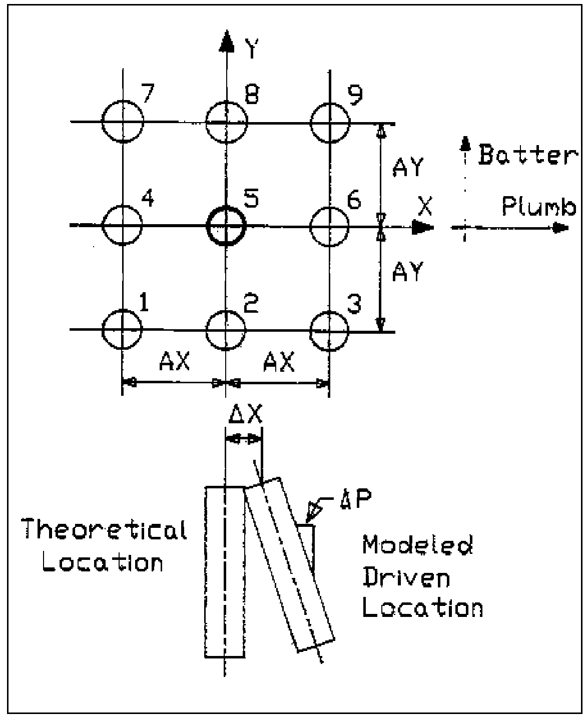


Figure 1-5. Typical nine-pile group for analysis

from the axis of the interior pile to the axis of any other pile is less than the pile diameter at any depth. If so, an intersection would occur for that particular combination of the 36 error values. The simulation is repeated for a large number of trials, each with newly generated random values for the error variables. The error values are generated such that, in the long run, the distribution of their values matches the assumed probability distributions. As the number of trials becomes large, the ratio of the number of trials with intersections to the total number of trials provides an increasingly accurate estimate of the probability of intersection.

d. Probability distribution for the error variables. The four random variables characterizing the pile placement and alignment errors, Δx, Δy, Δp, and Δb are assumed to fit the normal, or Gaussian, distribution found in most standard statistics books. This assumption is justifiable and convenient for a number of reasons:

(1) The normal distribution is bell-shaped and symmetrical. If the pile is assumed to be driven, on the average, at the theoretical location, then small deviations are more likely than large ones, and deviations are equally likely in either direction. These properties are consistent with expected construction practice.

(2) The normal distribution is commonly used to model random errors in a variety of systems; in fact, its development traces from error analysis.

(3) The normal distribution is completely defined by two parameters, the mean and standard deviations; if these are specified for the error variables, their entire distributions are defined and the probability of the variables assuming any set of values is readily calculated.

(4) Normally distributed random numbers are easily generated by simple computer algorithms.

The means of the four random error variables are taken as zero. This implies that, on the average, the piles are driven at the theoretical location and alignment. The standard deviations of the error variables, $\sigma_{\Delta x}$, $\sigma_{\Delta y}$, $\sigma_{\Delta b}$, and $\sigma_{\Delta p}$, define a measure of the scatter of the possible values about the mean. As the normal distribution extends to plus and minus infinity, the variables can assume any value; however, as illustrated in Figure 1-6, the values have a practical range of 3 to 4 standard deviations. For a normally distributed random variable, 68.27 percent of all values will lie within one standard deviation from the mean, 95.45 percent within 2 standard deviations, 99.73 percent within 3 standard deviations, and 99.994 percent within 4 standard deviations.

e. Default values for the standard deviation of the error variables. The software package CPGP provides default values for the standard deviation of the four error variables. If better data on the expected deviations are available, other values may be specified at the time of program execution. The standard deviations are used by the random number generator to scale the variation of the generated error values. The default values were selected to be consistent with both actual construction and normal tolerances. In a reasonably well-controlled manufacturing or production process aimed at producing products within a tolerance, the acceptable tolerances will typically correspond to bounds of two to three standard deviations from the mean value (Reference j). Assuming that this is the case and the pile deviations are normally distributed would imply

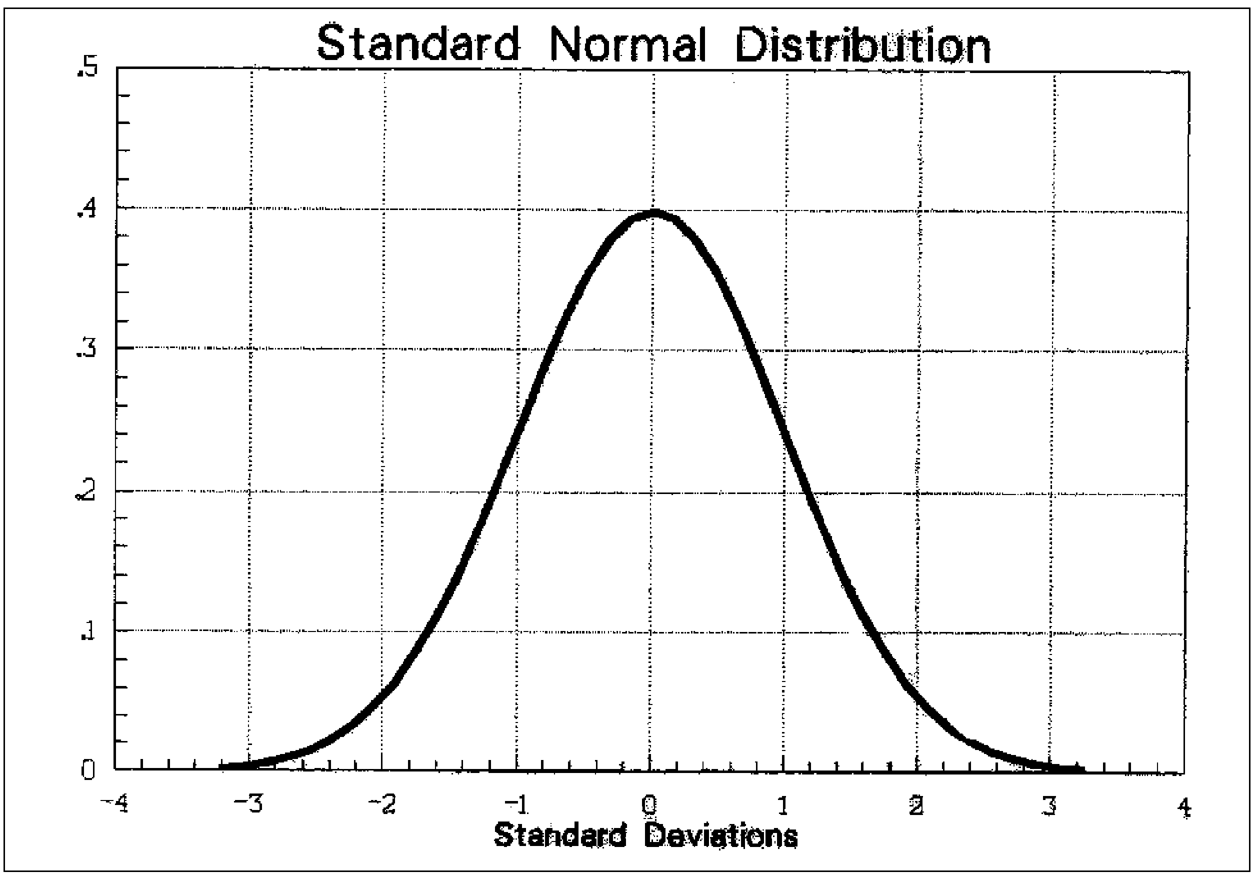


Figure 1-6. Standard normal distribution

that 4.55 percent (2σ) to 0.27 percent (3σ) of the piles will be driven out-of-tolerance and either rejected or erroneously approved, and specified tolerances should be met 95 to 99 percent of the time if "normal" pile driving practice is followed. To assign the default standard deviations for the error variables in the program, actual data on driven piles at Locks and Dam No. 26 (Replacement) were evaluated, and the standard deviations of the errors were compared to tolerances. The default values and their relation to usual tolerances are shown in Table 1-1. Using the above default values and tolerances, the program will generate location errors that exceed usual tolerance 4.55 percent of the time, batter errors that exceed usual tolerance 1.24 percent of the time, and plumb errors that exceed usual tolerance 9.70 percent of the time. If normal inspection procedures will detect and correct most cases where tolerances are exceeded, use of the default values should lead to a conservative estimate of intersection probability.

f. Program use. Detailed instructions for using the software package CPGP are contained in the user's guide (Reference h). The software package is written for IBM compatible PC's using the MS-DOS operating system and has the capability of modeling vertical or battered piles, and straight or cambered piles. The probability of intersection for a single interior pile is evaluated as a function of pile diameter, length, spacing, batter, and the standard deviations of the four error variables. Because of the iterative nature of the simulation program, and the fact that confidence limits on the solution are inversely proportional to the square root of the number of trials, running times can be relatively long (as much as two hours on an 80386 microcomputer with 80387 math coprocessor).

5. Chart Solutions

Due to the relatively long running time for the simulation program and its potentially infrequent use, a set of chart solutions has been prepared for vertical, uncambered piles using the default values for the random error variables and assuming that the pile spacing is equal in both directions. These charts are provided in Enclosure 3. For many cases, these charts will provide sufficient information to determine the probability of intersection for a single interior pile. Where piles are to be driven on a batter, or where different values for the standard deviations of the error variables are assumed, it is necessary to run the simulation.

6. Estimating the Probability Distribution for the Number of Intersections in a Pile Group

a. Equivalent number of interior piles. The probability of intersection for a typical interior pile is desired to obtain the expected number of intersections and the probability of 0, 1, 2, etc., intersections in a large group of piles. Special considerations must be made for the exterior and corner piles in a group. As an exterior pile has adjacent piles in only two of four quadrants, it is approximated by a statistically equivalent to one-half an interior. Likewise, the corner piles in a group are approximated by statistically equivalent of one-fourth an interior pile. While the approximations for the exterior and corners are not exact solution for probability distribution of these piles, they do provide sufficient accuracy for this application. Thus, the equivalent number of interior piles in a group can be taken as:

Table 1-1
Standard Deviation Versus Tolerance

Variable	Program Default Value for Standard Deviation	Usual Tolerance	Tolerance/Std. Dev.
Δx and Δy	$\sigma_{\Delta x}$=1.5 in.	3.0 in.	2.0
Δb	$\sigma_{\Delta b}$=0.10 in./ft	0.25 in./ft	2.5
Δp	$\sigma_{\Delta p}$=0.15 in./ft	0.25 in./ft	1.666

Interior piles + (1/2) exterior piles

+ (1/4) corner piles

For a group of m rows by n columns, this becomes:

$$(m-2)(n-2) + (1/2)(2)[(m-2)+(n-2)] + (1/4)(4)$$

Expanding and collecting terms, this becomes:

$$EIP = mn - m - n + 1$$

where EIP is the number of equivalent interior piles. For example, a 20 by 40 pile group would have 800 actual piles and 741 equivalent interior piles.

b. Expected number of intersections. The simulation model calculates the probability that the center pile of a nine pile group will intersect an adjacent pile. If a group of piles is considered, an intersection is not an independent event as every intersection involves two piles. However, a conservative estimate of the total probability of intersection can be obtained by assuming independence and employing the binomial distribution. This is analogous to assume that driving pile 18 into pile 21 is a different event than driving pile 21 into pile 18. In probability theory, the binomial distribution is used to predict the probability of the number of "successes" x, that will occur in a set of n independent trials when the probability of success is p for each trial. For the problem at hand, a pile intersection is considered a "success" in the probabilistic sense. The expected number of successes, or intersections, I, is given by:

$$E[I] = Np$$

where N is taken as the equivalent number of interior piles. Thus, for the example 20 x 40 pile group, if p has been previously found to be 0.002 for a single pile:

$$E[I] = (741)(.002) = 1.482$$

The expected value of 1.482 is the best estimate that can be made of the probable number of intersections. If a cost can be identified for the occurrence of an intersection, then the expected number of intersections times the cost per intersection represents the financial risk.

c. Probability distribution for the number of intersections. Although the expected number of intersections for this example is 1.482, the actual number of intersections must be a member of the set 0, 1, 2, ... According to the binomial distribution, the probability of x intersections is:

$$Pr(x) = \left(\frac{N!}{x!(N-x)!}\right) p^{x}(1-p)^{N-x}$$

Replacing x with I and continuing with the example, the binomial distribution gives:

$Pr(I=0) = 0.22685$ $Pr(I>0) = 1.0 - 0.22685 = 0.77315$
$Pr(I=1) = 0.33686$ $Pr(I>1) = 1.0 - 0.22685 - 0.33686 = 0.43629$
$Pr(I=2) = 0.24978$ $Pr(I>2) = 0.18831$
$Pr(I=3) = 0.12330$ $Pr(I>3) = 0.06501$
etc.

Thus, there is about a 23 percent chance of no intersections, a 77 percent chance of at least one intersection, a 43 percent chance of more than one intersection, a 19 percent chance of more than two intersections, and only a 6.5 percent chance of more than three intersections. Due to the factorials, the binomial distribution becomes unwieldy to calculate, even with a computer, for large values of N. It can be closely approximated using the Poisson distribution in the following form:

$$Pr(x) = \frac{(Np)^{x}}{x!} e^{-Np}$$

Again, x would be replaced with the number of intersections, I. The following tabulation indicates the similarity of the binomial and Poisson solutions for the case of a 20- by 40-pile group with p = 0.002.

No. of Intersections, I	Pr(I) (Binomial)	Pr(I) (Poisson)
0	0.22685	0.22718
1	0.33686	0.33669
2	0.24978	0.24948
3 or more	0.18651	0.18665

The software package CPGP provides a convenient means for calculating the distribution of the number of intersections using the Poisson distribution.

7. Limitations

The probabilistic approach to pile interference assessment is not a substitute for writing specifications that are as restrictive as necessary and enforcing them by adequate quality control and quality assurance procedures. In fact, the probabilistic procedure depends on such control and implies that the standard deviation of the actual alignment errors will not be greater than about one-third to one-half the specification tolerance. The procedure assumes rigid piles does not account for bending of battered piles that are inadequately supported or piles veering from a straight line due to obstructions.

8. Tolerable Probabilities

The use of a probabilistic procedure implies that some piles may intersect. If an intersection is believed to have occurred, the as-driven location and alignment should be measured to ensure that the specifications have been met. Then the piles should be pulled, inspected, replaced if necessary, and redriven. If one or more intersections go unnoticed, the intersecting piles may sustain structural damage and not provide the design capacity. For piles to be pulled, costs can be associated with pulling, redriving, and related delays. These costs can be multiplied by the expected number of intersections given in paragraph 6 to determine the expected intersection cost. The probability distribution for the expected intersection cost can be determined by multiplying the probability of 0, 1, 2, 3, etc., intersections by the associated costs of such intersections. As pile intersections may not always be apparent, a check should be made of the foundation capacity associated with 1, 2, 3, etc. random piles being damaged. In the absence of detailed cost and capacity studies, it would appear prudent to develop pile layouts such that the probability of intersection for single piles is less than about 0.002 and the expected number of intersections in a group is less than about 0.5.

9. Preliminary Findings

Experience with the probabilistic procedure is limited at this time. From the previous research, certain general conclusions can be drawn.

a. Pile length and spacing. The probability of intersection is sensitive to pile length. For common pile sizes and spacings, pile lengths shorter than 50 to 60 ft correspond to small probabilities of intersection, and lengths greater than 80 to 90 ft correspond to relatively large probabilities of intersection. Between these ranges, the probability of intersection increases two or more orders of magnitude.

b. Placement and alignment tolerances. The primary factor affecting pile intersection is the plumb and alignment tolerance, typically set at 0.25 in./ft. Variations in the standard deviation of the alignment error result in significant changes in the probability of intersection. Variations in the standard deviation of the ground placement error make much less difference. At 0.25 in./ft, the tip of a 100-ft long pile would deviate 25 in. from its theoretical location, which is over eight times greater than the 3 in. maximum deviation caused by ground placement. Thus, where intersection is of concern, efforts should be made to carefully inspect pile alignment in the field.

c. Pile camber and sweep. Standard specifications allow piles to deviate from perfect straightness (References c, d, h). The CPGP software allows the modeling of cambered and swept (curved) piles. The degree of camber and sweep on all simulated piles is taken to be that specified but the direction of curvature is simulated randomly. While it might be expected that cambered and swept piles would be more likely to intersect than straight piles, comparative analyses show a negligible difference. In a probabilistic model, curved piles are equally likely to curve away from each other as toward each other, and these effects tend to cancel out.

10. References

a. EM 1110-2-2906. "Design of Pile Foundations."

b. "Steel H-Piles." Apr 1990. Civil Works Construction Guide Specification CW-02315.

c. American Society for Testing and Materials. 1987. "Standard Specification for General Requirements for Rolled Steel Plates, Shapes, Sheet Piling, and Bars for Structural Use," ASTM A 6/A 6M-87d.

d. American Society for Testing and Materials. 1988. "Standard Specification for Round Timber Piles," ASTM D 25-88.

e. Fuller, F. M. 1983. *Engineering of Pile Installations*, McGraw-Hill Book Co., New York.

f. Hunt, H. 1979. "Design and Installation of Pile Foundations," Associated Pile and Fitting Corp., Clifton, NJ.

g. Lapin, L. L. 1983. "Probability and Statistics for Modern Engineering," Brooks/Cole Publishing Co. Division of Wadsworth, Inc., Belmont, CA.

h. PCI Committee on Prestressed Concrete Piling. Mar-Apr 1977. "Recommended Practice for Design, Manufacture, and Installation of Prestressed Concrete Piling," *Journal of the Prestressed Concrete Institute*, Vol 22, No. 2.

i. Wolff, Thomas F. Apr 1990. "PILINT: A Software Package Designed for Pile Interference Analysis," research report prepared for US Army Corps of Engineers, Michigan State University.

j. Wolff, Thomas F. Sep 1990. "User's Guide: Pile Group Interference Probabilistic Assessment (CPGP) Computer Program," contract report to US Army Engineer Waterways Experiment Station.

PILE LAYOUT TO MINIMIZE INTERFERENCE
SAMPLE SPECIFICATION

The following specification wording is suggested to deal with pile intersection.

> <u>Pile Intersection</u>: *If pile driving conditions indicate that a driven pile may have intersected another pile, the Contracting Officer shall immediately be notified. If the Contracting Officer believes that an intersection may have occurred, he may at his option direct the Contractor to survey the location and alignment of the piles, pull the piles, furnish new piles, redrive the same piles, or drive new piles. If a pulled pile was driven within tolerances, full payment will be made to the Contractor for pulling the pile, furnishing a new pile (if required) and redriving the pile at the applicable unit prices for pulling, furnishing, and redriving piles. If the pile was initially driven out of the specified tolerances, pulling, redriving and furnishing a new pile shall be done by the Contractor at no cost to the Government.*

PILE LAYOUT TO MINIMIZE INTERFERENCE CHART SOLUTIONS

Figures 3-1 through 3-12 provide chart solutions for the probability of intersection of a single interior pile as a function of length and spacing for pile diameters from 10 to 24 in. Piles are assumed to be vertical, equally spaced in the x and y directions, and have zero camber. These charts were developed using CPGP with the default values for the standard deviation of the error variables:

$\sigma_{\Delta x}$ = 1.5 in.

$\sigma_{\Delta y}$ = 1.5 in.

$\sigma_{\Delta p}$ = 0.15 in./ft

$\sigma_{\Delta b}$ = 0.10 in./ft

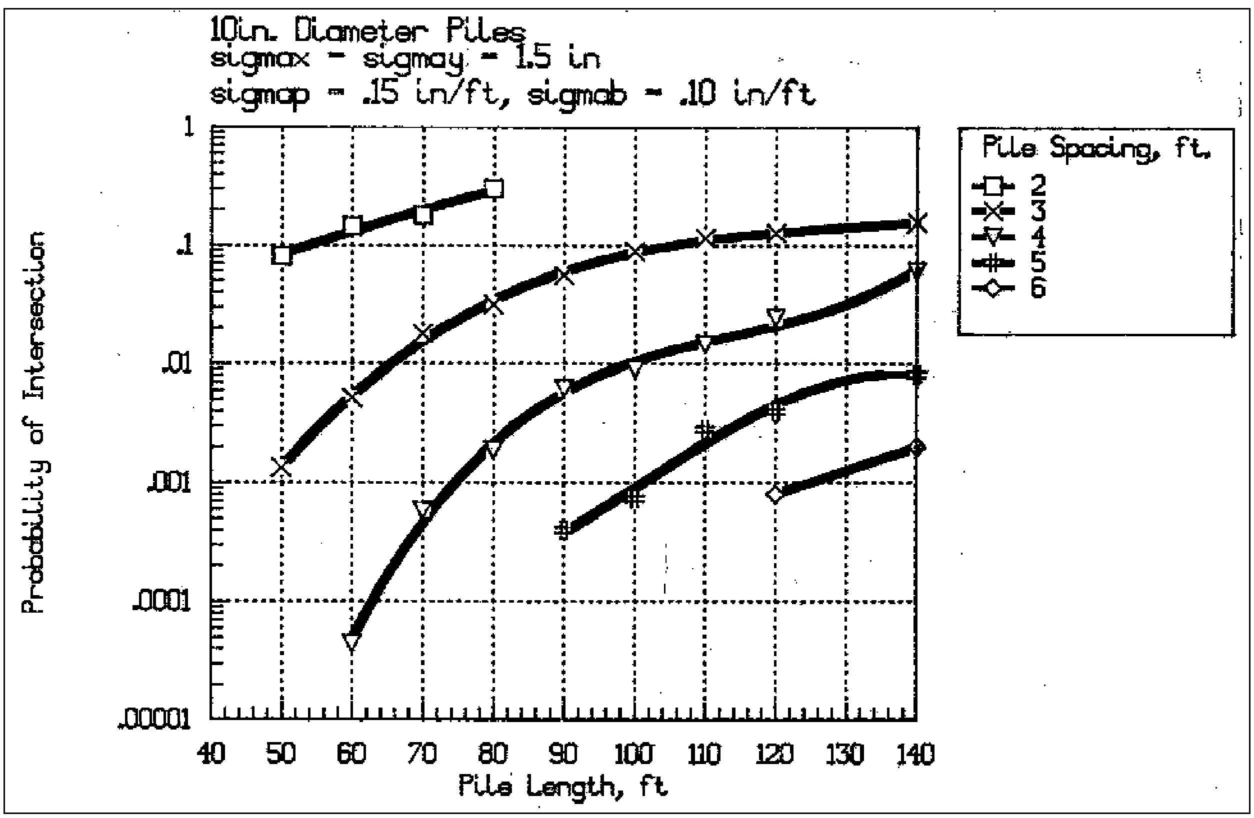


Figure 3-1. Probability of intersection versus length, 10-in. piles

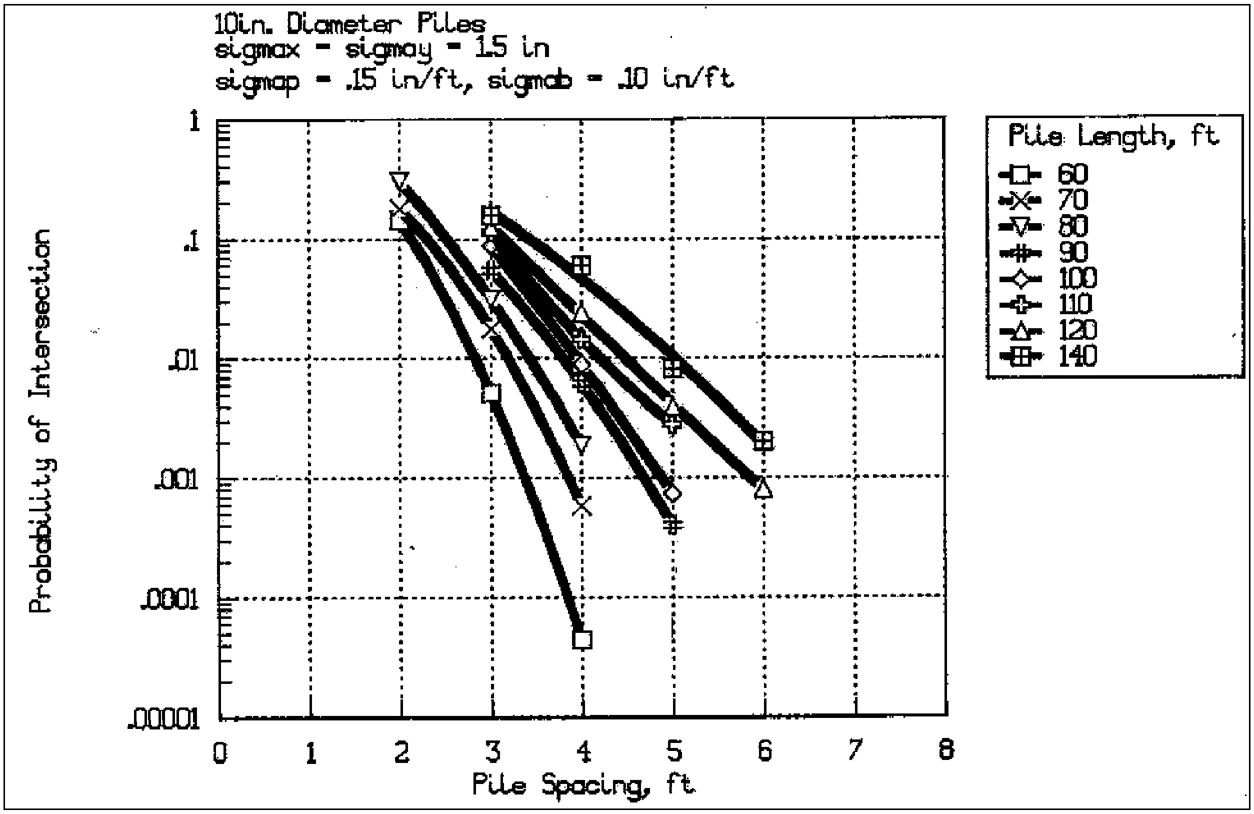


Figure 3-2. Probability of intersection versus spacing, 10-in. piles

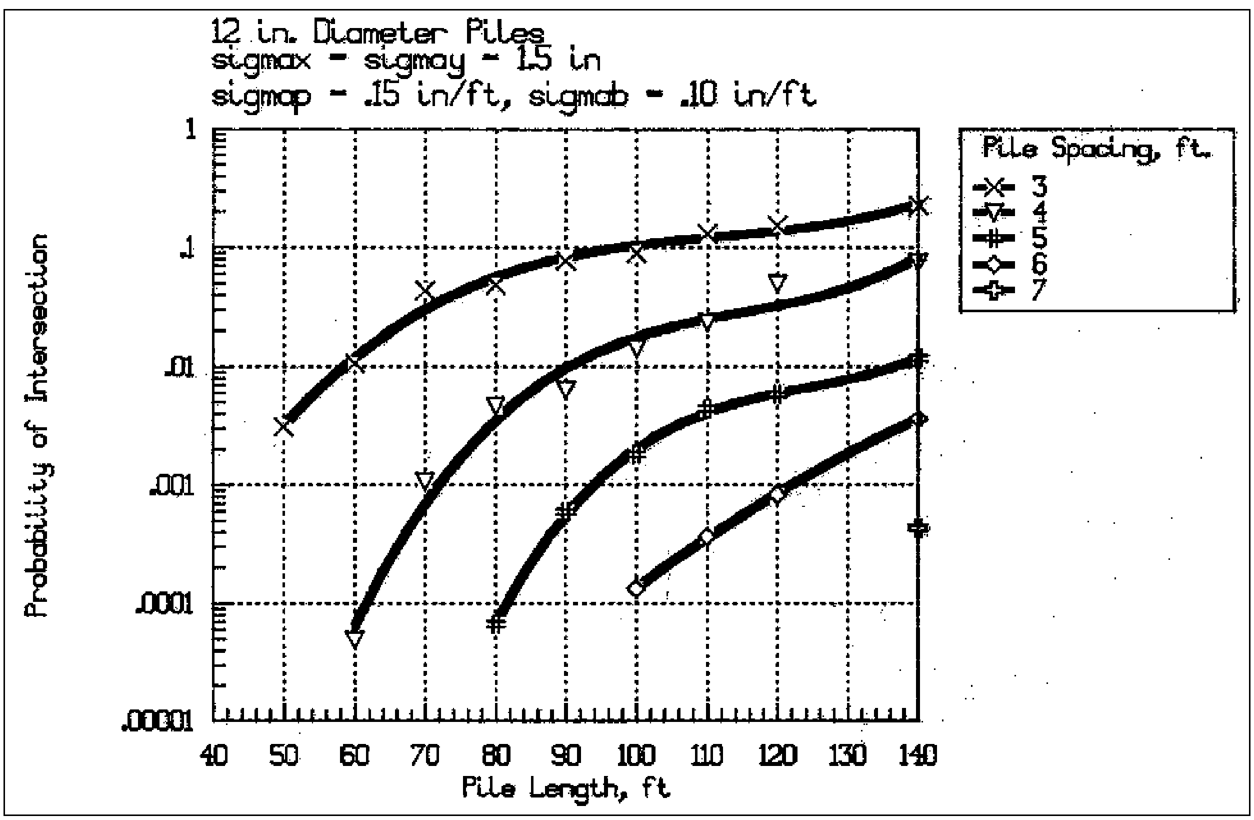


Figure 3-3. Probability of intersection versus length, 12-in. piles

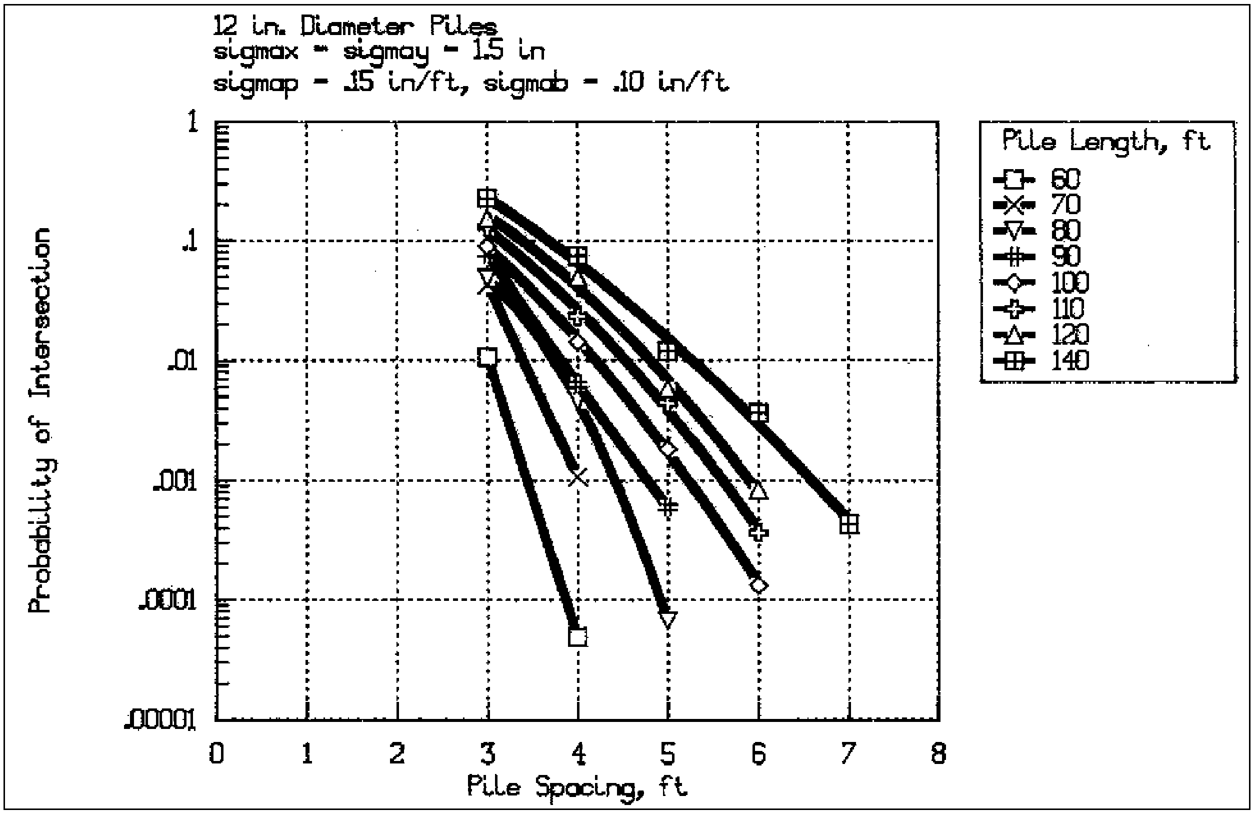


Figure 3-4. Probability of intersection versus spacing, 12-in. piles

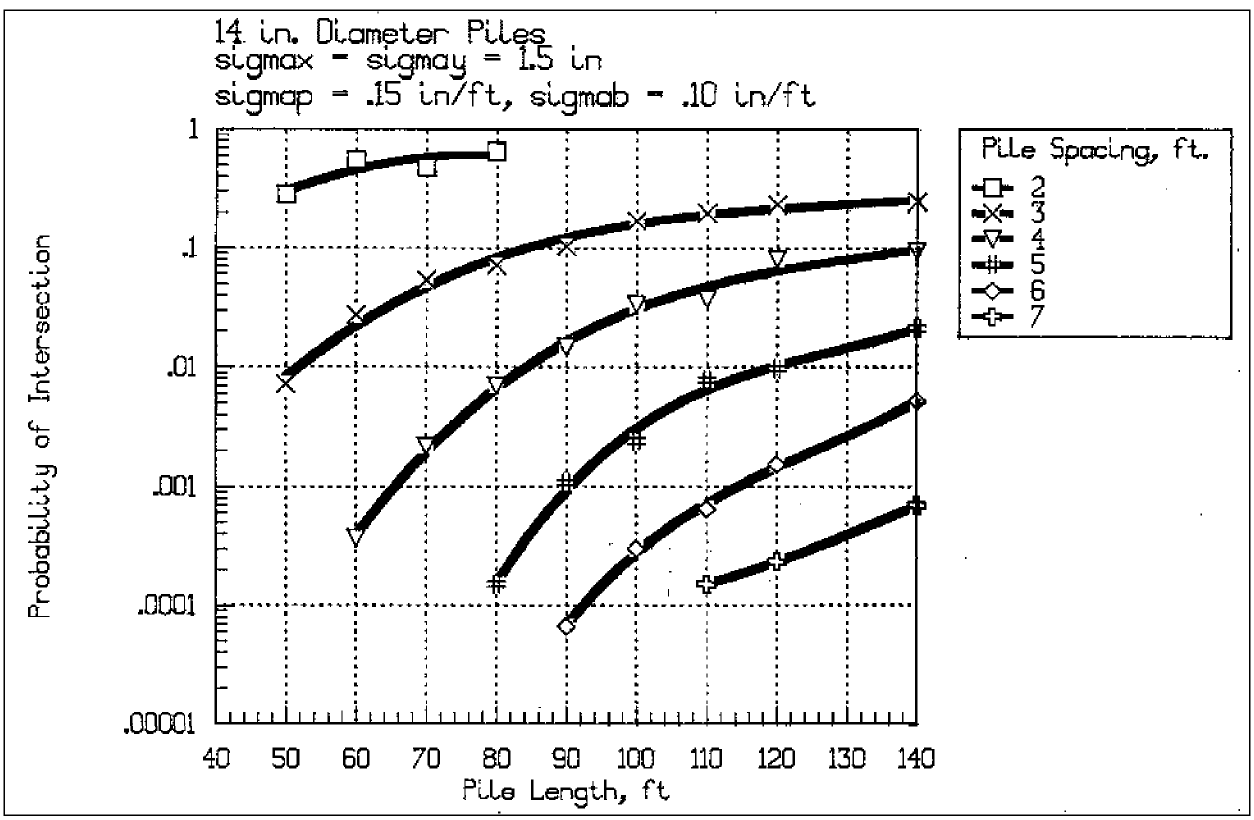


Figure 3-5. Probability of intersection versus length, 14-in. piles

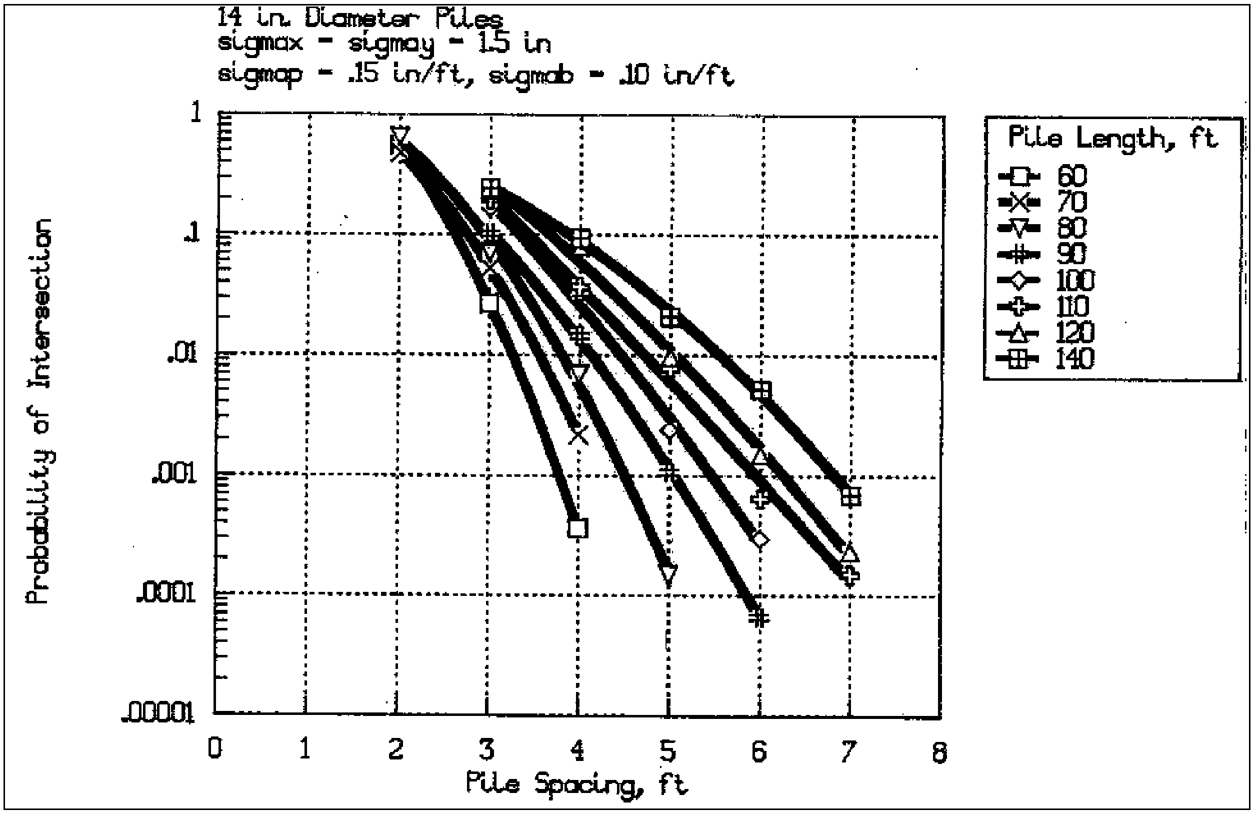


Figure 3-6. Probability of intersection versus spacing, 14-in. piles

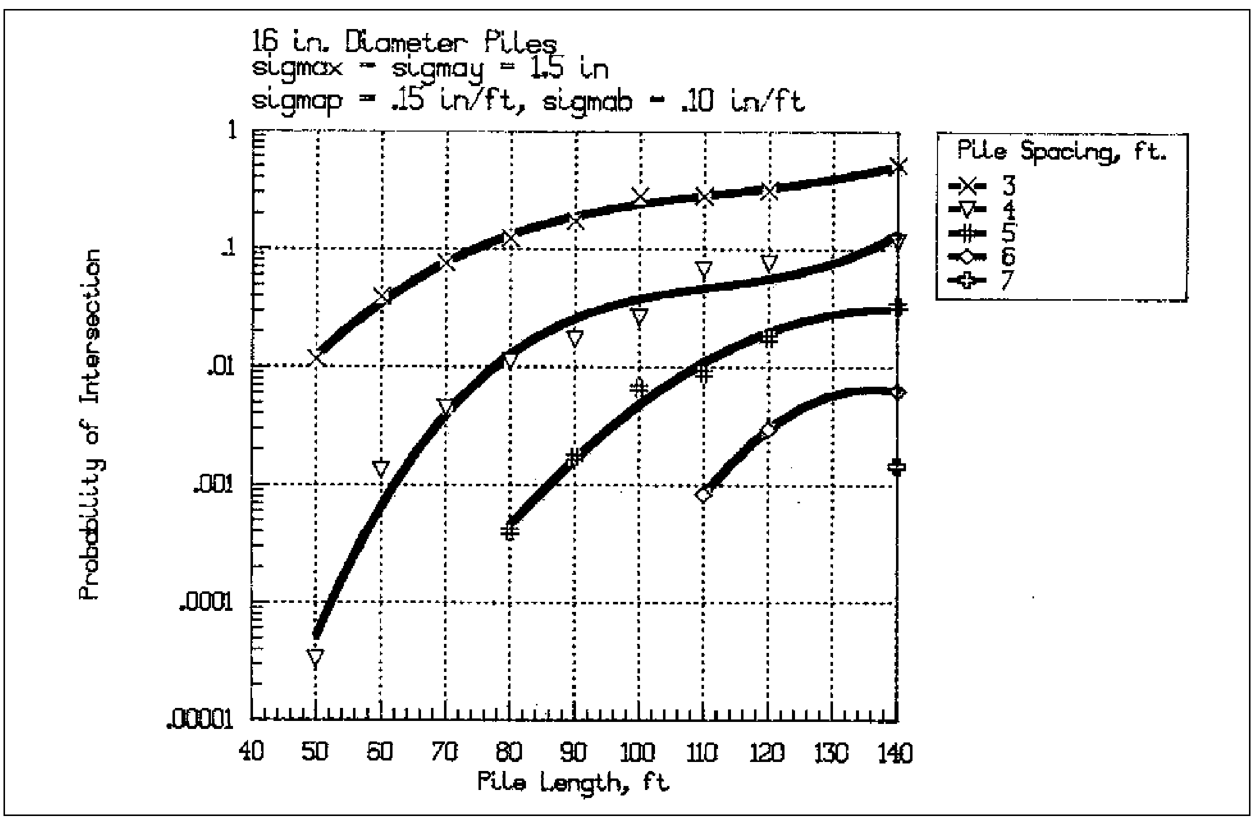


Figure 3-7. Probability of intersection versus length, 16-in. piles

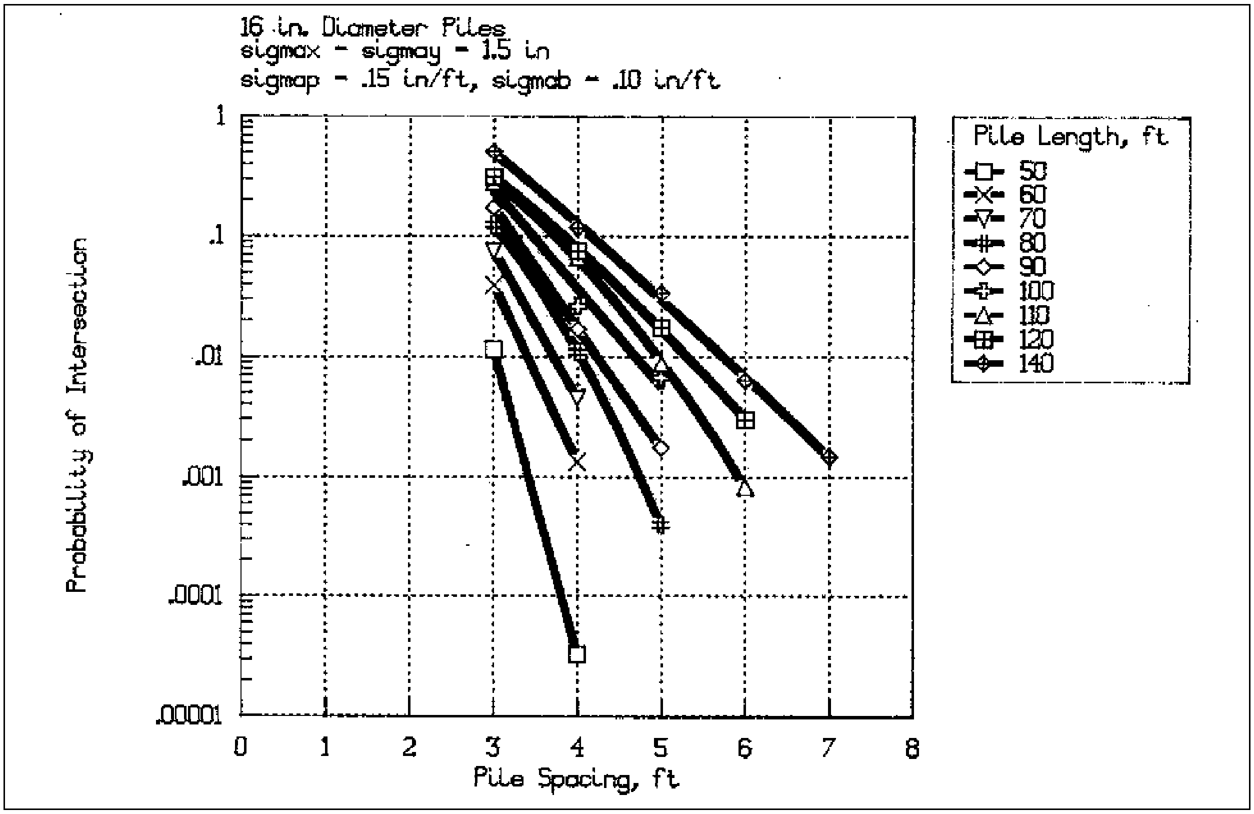


Figure 3-8. Probability of intersection versus spacing, 16-in. piles

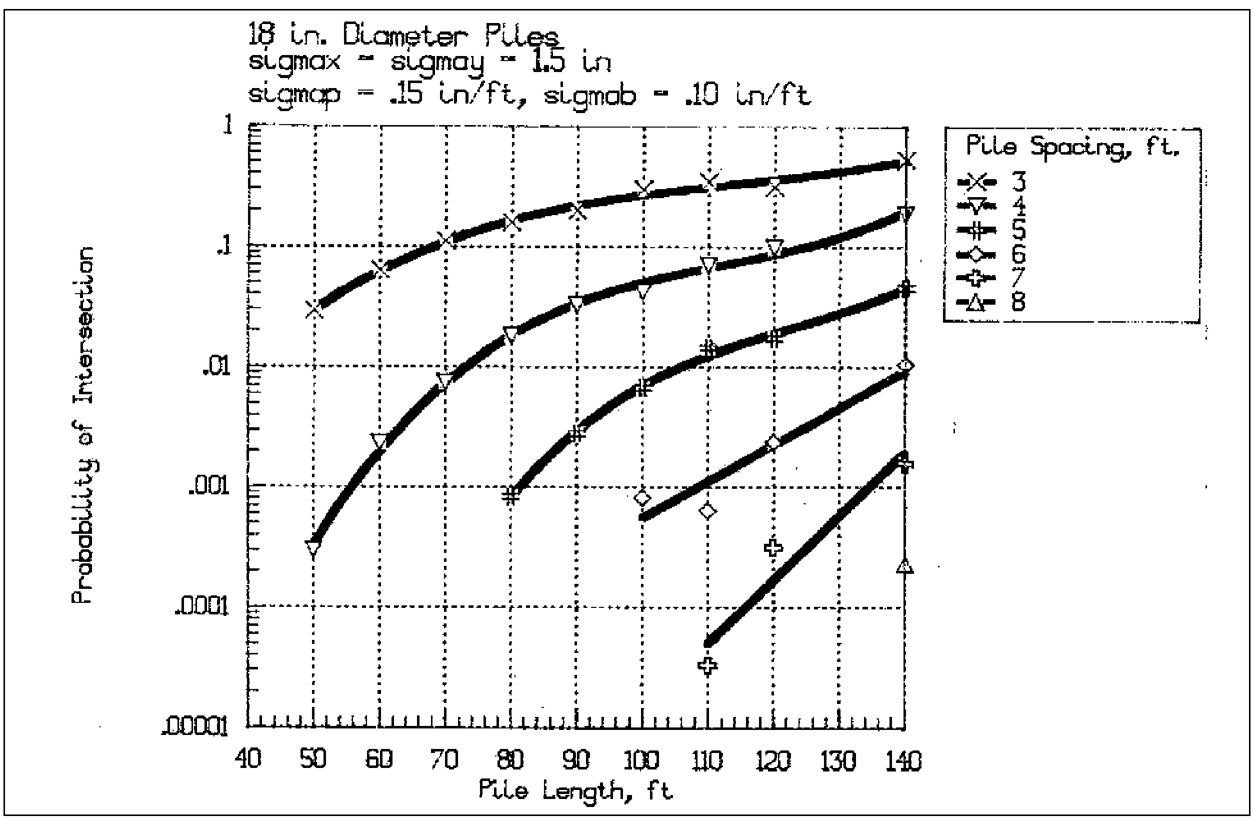


Figure 3-9. Probability of intersection versus length, 18-in. piles

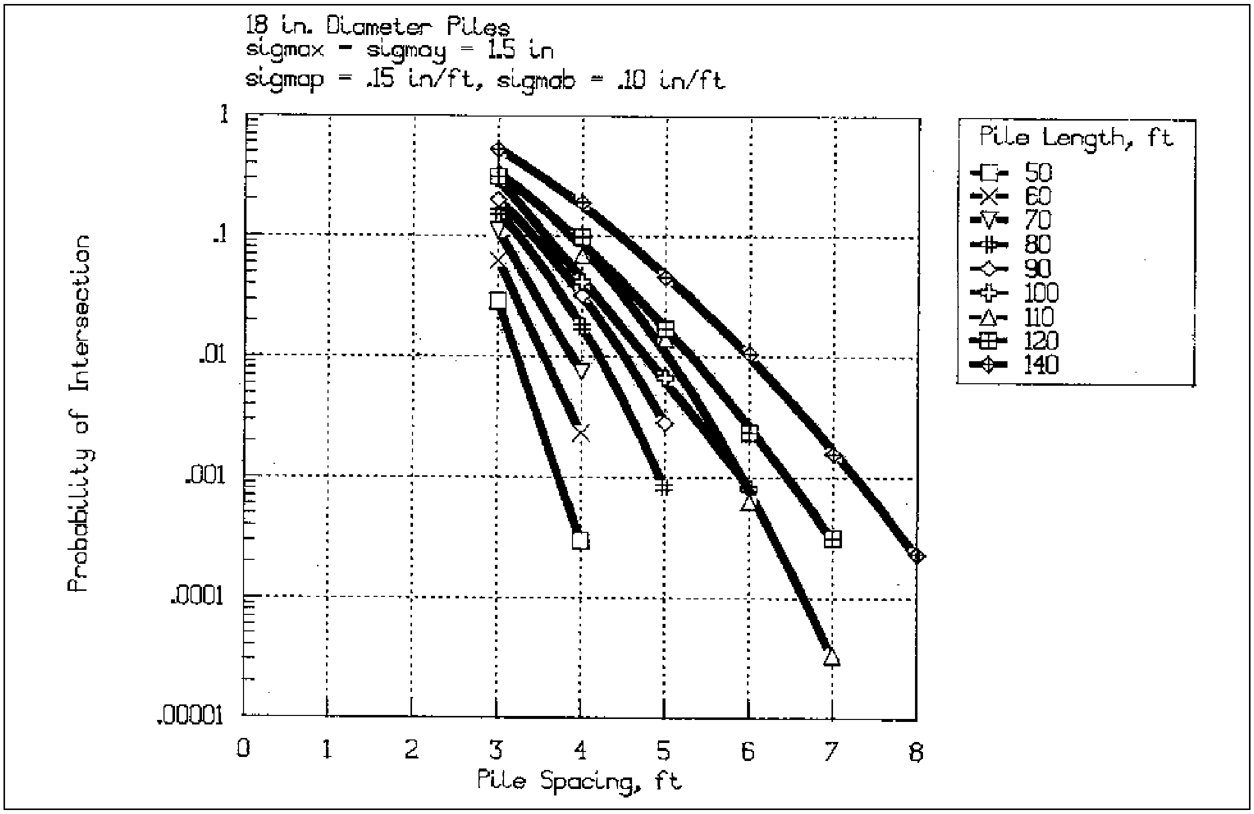


Figure 3-10. Probability of intersection versus spacing, 18-in. piles

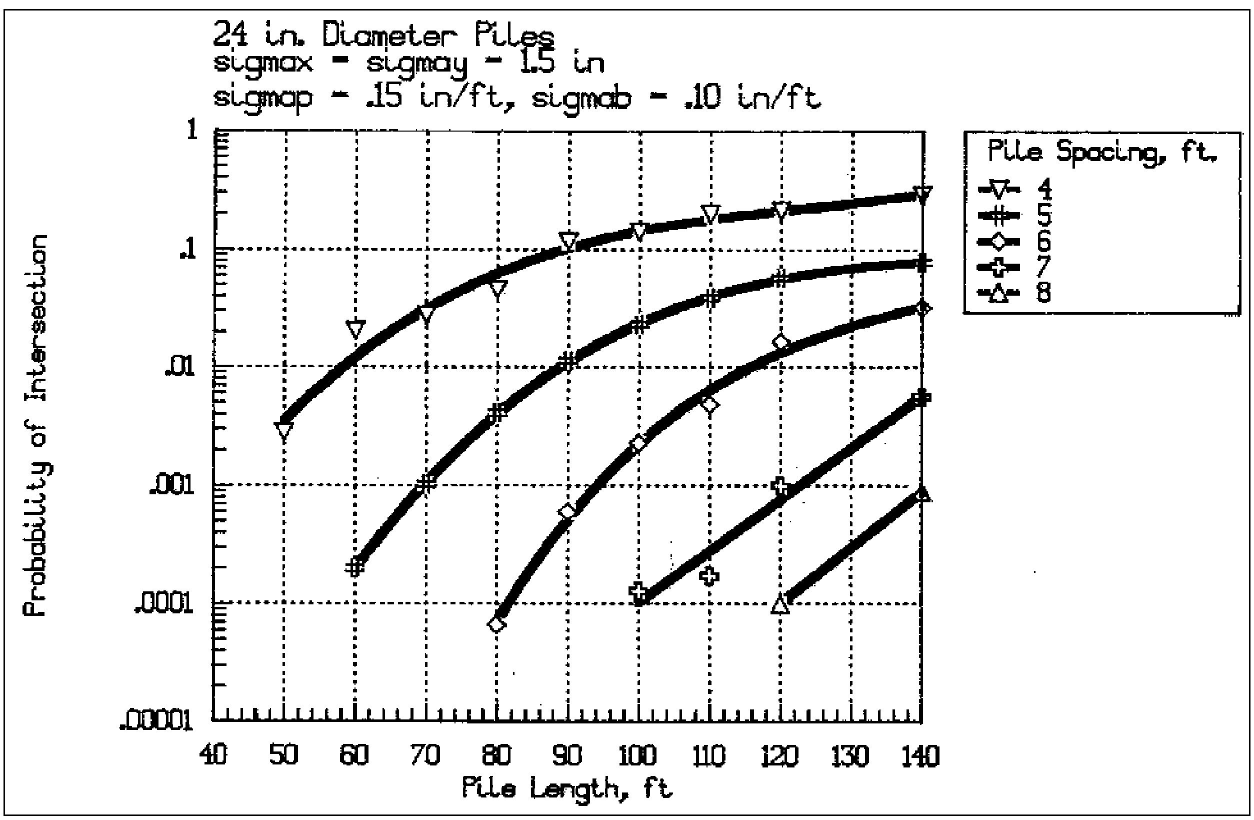


Figure 3-11. Probability of intersection versus length, 24-in. piles

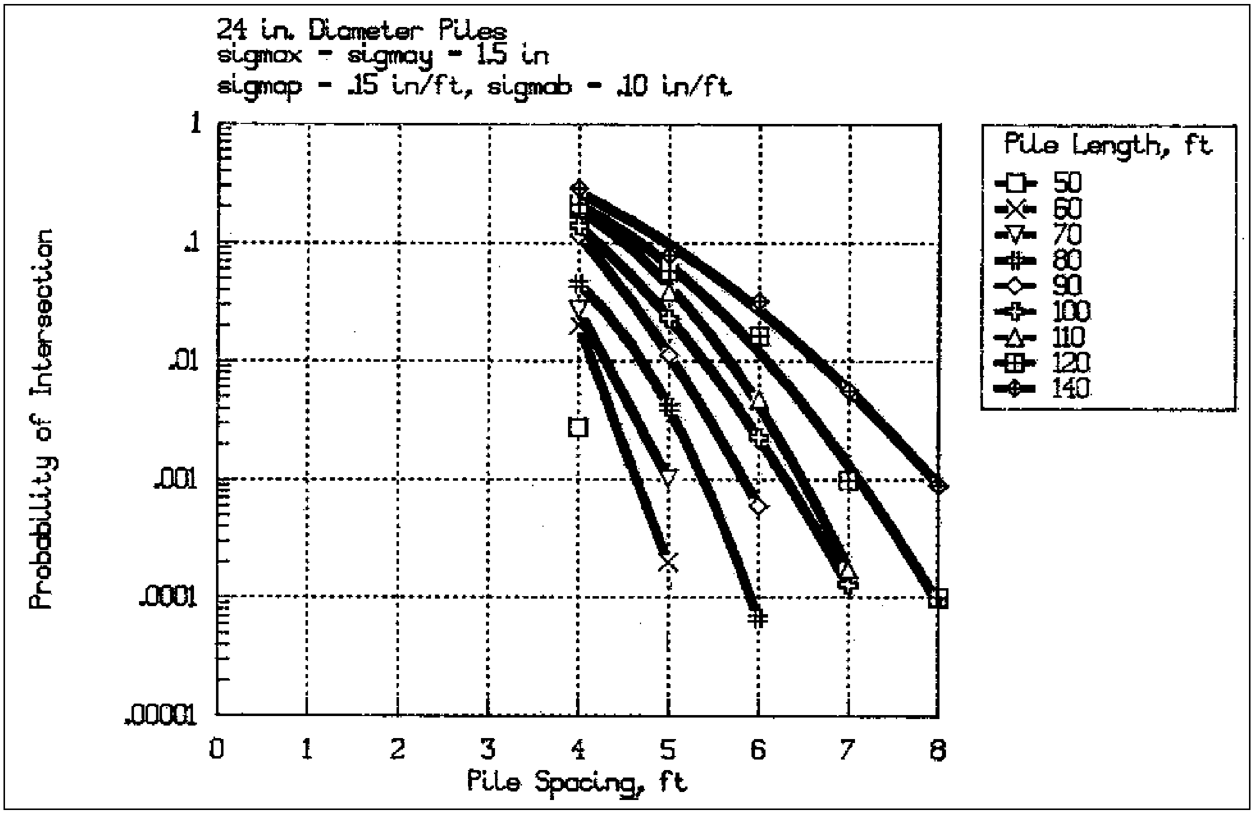


Figure 3-12. Probability of intersection versus spacing, 24-in. piles

PILE LAYOUT TO MINIMIZE INTERFERENCE

EXAMPLE 1
PILE LENGTH, SPACING, AND DIAMETER OPTIONS
PILE IN UNIFORM CLAY

1. Description

This example illustrates the evaluation of design alternatives for a pile group in a uniform clay. Eight alternative designs are developed and the probability of intersection, expected number of intersections, and comparative costs and risks of intersection are determined for each design.

2. Design Requirements and Static Analysis

Assume a group of steel pipe piles are to be driven in a uniform clay. The piles are to support a load of 7 kips/sq ft over a 20-ft square area and provide a factor of safety of 2.5. Thus, the required ultimate capacity of the group is 7.00 × 20 × 20 × 2.5 = 7,000 kips. The static pile capacity is to be determined by the α method (EM 1110-2-1906). The undrained strength (or cohesion), s_u of the clay is 1,000 lb/sq ft and the skin resistance, f ($f = \alpha s_u$), is 750 lb/sq ft.

The ultimate pile capacity for a single pile, Q_{ult}, is:

$$Q_{ult} = Q_{side} + Q_{tip}$$

$$Q_{ult} = fpL + 9 s_u A_{tip}$$

Where

Q_{side} is the ultimate side or shaft resistance
Q_{tip} is the ultimate tip or point resistance
p is the perimeter of the pile
L is the embedded length of the pile
A_{tip} is the cross-sectional area of the pile tip

The capacity of the group is the lesser of the capacity of a single pile times the number of piles,

$$Q_{group} = nQ_{ult}$$

or the capacity of the entire group failing as a unit:

$$Q_{group} = p_{group} L_{group} \alpha s_u + N_c s_u A_{group}$$

where

n is the number of piles in the group
p_{group} is the perimeter of the group
L_{group} is the pile length, or embedded depth of the group
N_c is a bearing capacity factor between 5.14 and 9, depending on the width to depth ratio of the group
A_{group} is the base area of the group

It is assumed that piles can be spaced on 4- or 5-ft centers; thus, the group can consist of 25 piles spaced on 5-ft centers as shown in Figure 4-1 or

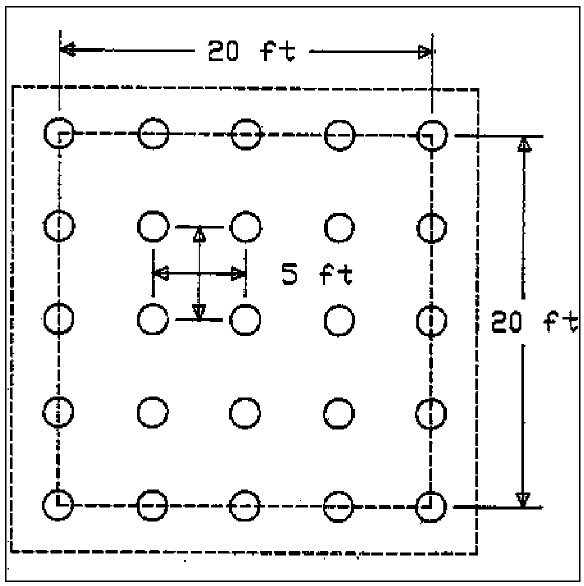


Figure 4-1. Twenty-five piles on 5-ft centers

36 piles spaced on 4-ft centers as shown in Figure 4-2. It is further assumed that piles of 12, 14, 16 and 18 in. diameters can be used. By calculating the required pile length to provide 7,000 kips ultimate capacity, eight comparative pile designs were developed using a microcomputer spreadsheet; an example printout from the spreadsheet is shown

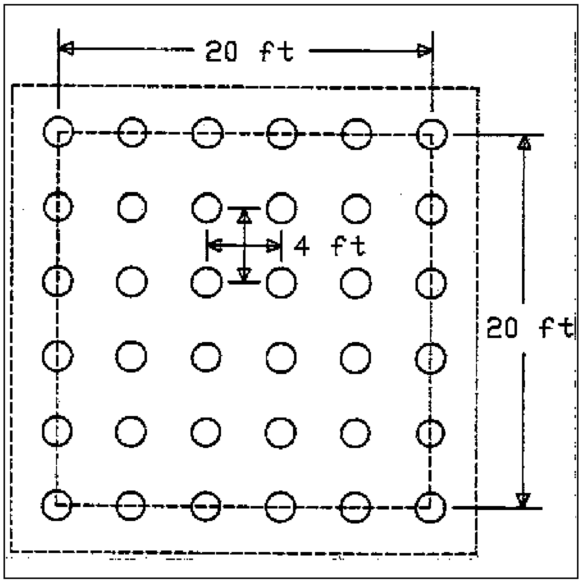


Figure 4-2. Thirty-six piles on 4-ft centers

in Figure 4-3. The resulting designs are tabulated below:

A: 36 piles, 4 ft spacing, 12 in. diameter, 80 ft long, Q_{ult} = 7,040 kips
B: 36 piles, 4 ft spacing, 14 in. diameter, 70 ft long, Q_{ult} = 7,274 kips
C: 36 piles, 4 ft spacing, 16 in. diameter, 60 ft long, Q_{ult} = 7,238 kips
D: 36 piles, 4 ft spacing, 18 in. diameter, 55 ft long, Q_{ult} = 7,570 kips
E: 25 piles, 5 ft spacing, 12 in. diameter, 120 ft long, Q_{ult} = 7,245 kips
F: 25 piles, 5 ft spacing, 14 in. diameter, 100 ft long, Q_{ult} = 7,113 kips
G: 25 piles, 5 ft spacing, 16 in. diameter, 90 ft long, Q_{ult} = 7,383 kips
H: 25 piles, 5 ft spacing, 18 in. diameter, 75 ft long, Q_{ult} = 7,024 kips

Each of the designs A through H will provide an ultimate capacity of just over 7,000 kips; however, each will have a different settlement, a different probability of intersection, a different cost, and a different financial risk attributable to possible intersection. The settlement calculations are beyond the scope of this example; however, everything else being equal, the designs with the greatest pile lengths (and hence the smaller diameter piles) will have the least settlement. Probability of intersection and cost considerations are discussed in the next section.

3. Probability of Intersection

From the published chart solutions, the probabilities of intersection for individual piles were determined for each of the eight designs. Using these probabilities and the group layouts, the probability distribution on the number of intersections was determined using the software package CPGP. The results are shown in Table 4-1.

The probability that one or more intersections will occur varies from about 10 percent for design A to about one-half of 1 percent for design H, or a twentyfold difference. The lowest probability of intersection occurs for the greater pile spacing, largest diameter pile, and a relatively short pile length.

4. Financial Risk

Consideration of the expected cost of possible intersections may provide a quantitative perspective to aid in making design decisions. Representative unit cost data for this example were provided by the Cost Engineering Section of the St. Louis District. Material costs were assumed to vary from $20.00/ft for 12-in. piles to $29.00/ft for 18-in. piles. Equipment and labor was taken at $530/hr. A set of productivity curves was provided giving the number of piles driven per hour as a function of pile length and diameter. Using these data, the comparative costs were determined assuming no intersections occur (Table 4-2).

For this example, the cost of an intersection is taken to be an additional cost equal to twice the furnishing and driving cost times two piles, as the piles must be both pulled and redriven, requiring two additional setups. There may be significant additional delay costs; these are assumed to be a flat $2,000.00 per intersection for the purpose of this example. The financial risk due to the possibility of intersection is the expected number of

```
|   A   ||   B   ||   C   ||   D   ||   E   ||   F   ||   G   ||   H   |
Pile Capacity for Friction Pile in Uniform Clay
T.F. Wolff and T.J. Mixter, 19 July 1990


Undrained Strength on Side,     su =  1000.00      psf
adhesion factor,             alpha =      .75
skin friction,                   f =   750.00      psf

Undrained Strength at Tip       su =  1000.00      psf
Tip Bearing                9 * su =   9000.00      psf


                      12 in    14 in    16 in    18 in
    L, ft           Q, kips  Q, kips  Q, kips  Q, kips
------------------------------------------------------------------------
    50.00            124.88   147.07   169.65   192.62
    60.00            148.44   174.55   201.06   227.96
    70.00            172.00   202.04   232.48   263.30
    80.00            195.56   229.53   263.89   298.65
    90.00            219.13   257.02   295.31   333.99
   100.00            242.69   284.51   326.73   369.33
   110.00            266.25   312.00   358.14   404.68
   120.00            289.81   339.49   389.56   440.02


CALCULATING THE GROUP EFFICIENCY OF PILES
GIVEN A GROUP OF PILES DIMENSIONED n1 X n2
IN AN AREA Bg X Lg

     n1 =        6                          <----- Lg ----->
     n2 =        6                     /\   o    o    o    o
     Bg =    21.00 (feet)               |                        /\
     Lg =    21.00 (feet)              Bg   o    o    o    o   |
      d =       12 (inches)             |                        n1
      c =     1000 (psf)               \/   o    o    o    o   _
      D =    80.00 (feet)
  alpha =      .75                          | n2 --->
spacing =     4.00

The group capacity is the lesser of the following two equations:

Equation 1:

   Capacity = n1 * n2 * ( Qt + Qs ) = N * ( Qt + Qs )

         where Qt = At * ( 9 * c )
               Qs = ( fs * As )

                  where fs = Ca = alpha * c
                        As = area of pile in contact with soil

   Capacity =          7040 (kips)


Equation 2:

   Capacity = 2 * ( Bg + Lg ) * D * Cav
              + [5 * (1 + D/ 5/ Bg) * (1 + Bg/ 5/ Lg)] * Cb * Lg * Bg
                  where Cav = alpha * c
                  Nc(calc)= 5 * ( 1 + D / 5 /Bg) * ( 1 + Bg / 5 / Lg )
                  Nc(calc)=    10.57          (maximum = 9)
                       Nc =        9

   Capacity =          9009 (kips)

```

Figure 4-3. Example of spreadsheet calculation of group capacity

Table 4-1
Probability Distribution

Design	Spacing ft	Diameter in.	L ft	Pr[I] (pile)	E[I] (group)	Pr[I =0] (group)	Pr[I >0] (group)
A	4	12	80	.004	.1	.9048	.0952
B	4	14	70	.002	.05	.9512	.0488
C	4	16	60	.0006	.015	.9851	.0149
D	4	18	55	.0008	.02	.9802	.0198
E	5	12	120	.005	.08	.9231	.0769
F	5	14	100	.003	.048	.9531	.0469
G	5	16	90	.0015	.024	.9763	.0237
H	5	18	75	.0003	.0048	.9952	.0048

Table 4-2
Comparative Costs

Design	Number of Piles	Pile Diam. in.	Pile Length ft	Driving Time piles/hr	Driving Costs $/ft	Material Costs $/ft	Total Unit Cost $/ft	Total Cost $
A	36	12	80	1.00	$ 6.63	$ 20.00	$ 26.63	$ 76,680
B	36	14	70	1.11	$ 6.82	$ 23.00	$ 29.82	$ 75,149
C	36	16	60	1.25	$ 7.07	$ 25.00	$ 32.07	$ 69,264
D	36	18	55	1.03	$ 9.36	$ 29.00	$ 38.36	$ 75,957
E	25	12	120	.66	$ 6.68	$ 20.00	$ 26.68	$ 80,041
F	25	14	100	.72	$ 7.36	$ 23.00	$ 30.36	$ 75,910
G	25	16	90	.69	$ 8.53	$ 25.00	$ 33.53	$ 75,441
H	25	18	75	.60	$ 11.73	$ 29.00	$ 40.73	$ 76,365

intersections times the cost of an intersection. The total expected costs of the alternative designs are thus:

Base cost + E[I] x (4 x L x $/ft + delay cost)

Design A $76,680 + 0.1000 x (4 x 80 x $26.63/ft + $2,000) = $77,732
Design B $75,149 + 0.0500 x (4 x 70 x $29.82/ft + $2,000) = $75,666
Design C $69,264 + 0.0150 x (4 x 60 x $32.07/ft + $2,000) = $69,409
Design D $75,957 + 0.0200 x (4 x 55 x $38.36/ft + $2,000) = $76,166
Design E $80,041 + 0.0800 x (4 x 120 x $26.68/ft + $2,000) = $81,226
Design F $75,910 + 0.0480 x (4 x 100 x $30.36/ft + $2,000) = $76,588
Design G $75,441 + 0.0015 x (4 x 90 x $33.53/ft + $2,000) = $75,462
Design H $76,365 + 0.0003 x (4 x 75 x $40.73/ft + $2,000) = $76,369

The resulting costs are plotted as a function of pile diameter in Figure 4-4. The solid curves are the total direct costs, and the dotted curves are the total expected costs including the expected cost due to interference. The two curves provide the designer a visual characterization of the financial risk of intersection. It is noted that the expected cost difference due to interference is greatest for the designs utilizing 12 in. piles, where the greatest pile lengths are

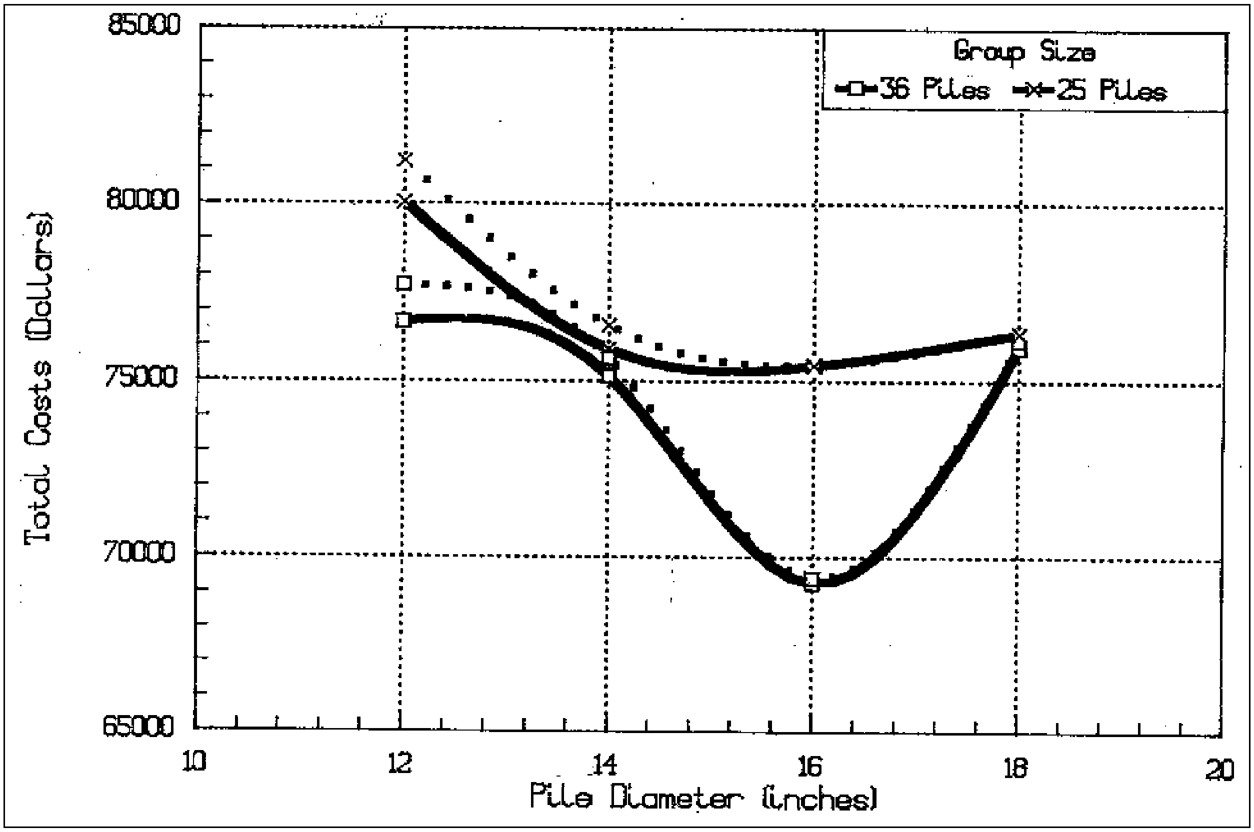


Figure 4-4. Comparative pile group costs

required, and are virtually negligible for the designs utilizing 18 in. piles, where the comparatively short pile lengths are associated with very small intersection probabilities. While the cost differences are not great in this particular example, they could differ greatly if the delay costs caused by intersection were higher.

PILE LAYOUT TO MINIMIZE INTERFERENCE

EXAMPLE 2
EFFECT OF PLACEMENT AND ALIGNMENT TOLERANCES ON INTERSECTION PROBABILITY

1. Purpose

The purpose of this example is to illustrate the possible effect of varying placement and alignment tolerances on the probability of intersection for a single pile.

2. Placement Tolerances

The program default standard deviation values for the pile butt ground location errors, $\sigma_{\Delta x}$ and $\sigma_{\Delta y}$ (SIGMAX and SIGMAY in the program CPGP), are 1.5 in. each. As the typical tolerance for pile butt location is 3.0 in., the normal tolerance corresponds to two standard deviations. The effect of changing the standard deviations over the range 0.5 in. to 3 in. was evaluated by making a parametric study involving 18 runs of the program. If the tolerances are assumed to correspond to two standard deviations for each case, implying that contractor tightens or relaxes control over pile placement consistent with the specified tolerances, then the parametric study reflects placement tolerances of 1.0 to 6.0 in. The results of the study are shown in Figure 5-1 where the probability of intersection is plotted against SIGMAY for three values of SIGMAX. The heavy line in the figure corresponds to the case of equal standard deviations in both directions. All points were obtained using the program default values for other parameters; i.e., a 14-in. diameter pile 80 ft long, with SIGMAP = 0.15 in./ft and SIGMAB = 0.10 in./ft. For the cases analyzed, it is observed that there is a slight increase in the probability of intersection with increasing standard deviation, but the variation is within one-half an order of magnitude. The example suggests that the probability of intersection is not greatly affected by the degree of precision in the ground location; as the standard deviations (and possibly tolerances) increase, there is an increasing chance of the piles being both closer together and further apart.

3. Alignment Tolerances

The program default standard deviation values for the pile alignment errors, $\sigma_{\Delta p}$ and $\sigma_{\Delta b}$ (SIGMAP and SIGMAB in the program), are 0.15 in./ft and 0.10 in./ft, respectively. As the typical tolerances for pile plumb and batter are 0.25 in./ft, the normal tolerances correspond to 1.67 and 2.5 standard deviations, respectively. The effect of changing the standard deviations over the range 0.01 to 0.35 in./ft was evaluated by making a parametric study involving 21 runs of the program. The results of the study are shown in Figure 5-2 where the probability of intersection is plotted against SIGMAB for four values of SIGMAP. The heavy line in the figure corresponds to the case of equal standard deviations in both directions. All points were obtained using the program default values for other parameters; i.e., a 14-in. diameter pile 80 ft long, with SIGMAX = SIGMAY = 1.5 in. It is observed that when the standard deviations are assumed to be equal, the probability of intersection is very sensitive to the standard deviation of the alignment error, varying almost three orders of magnitude as the standard deviations are varied from 0.01 in./ft to 0.35 in./ft. Thus, the probability of intersection is implicitly sensitive to the alignment tolerance and quality of inspection of the vertical alignment. This example suggests that the probability of intersection can be greatly affected by the degree of precision in setting and checking the pile verticality or deviation from theoretical batter.

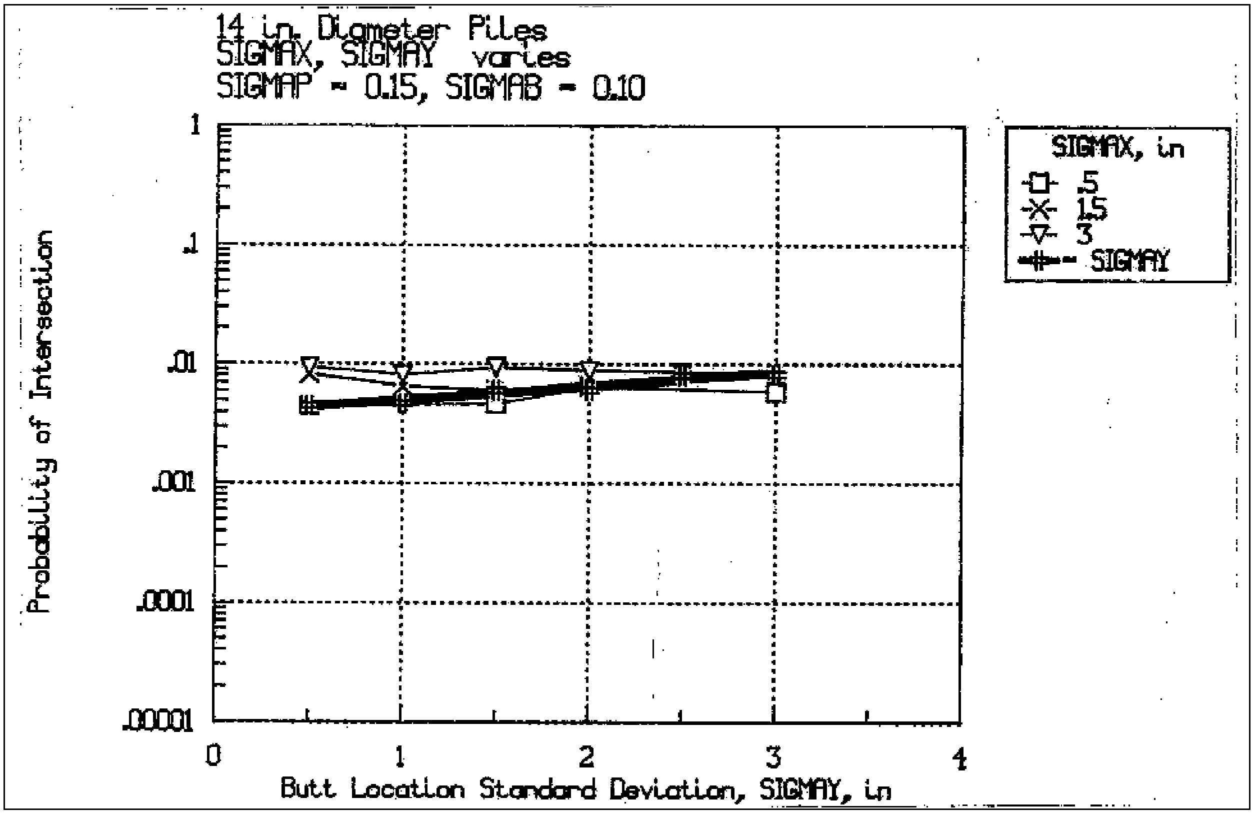


Figure 5-1. Effect of placement error on intersection probability

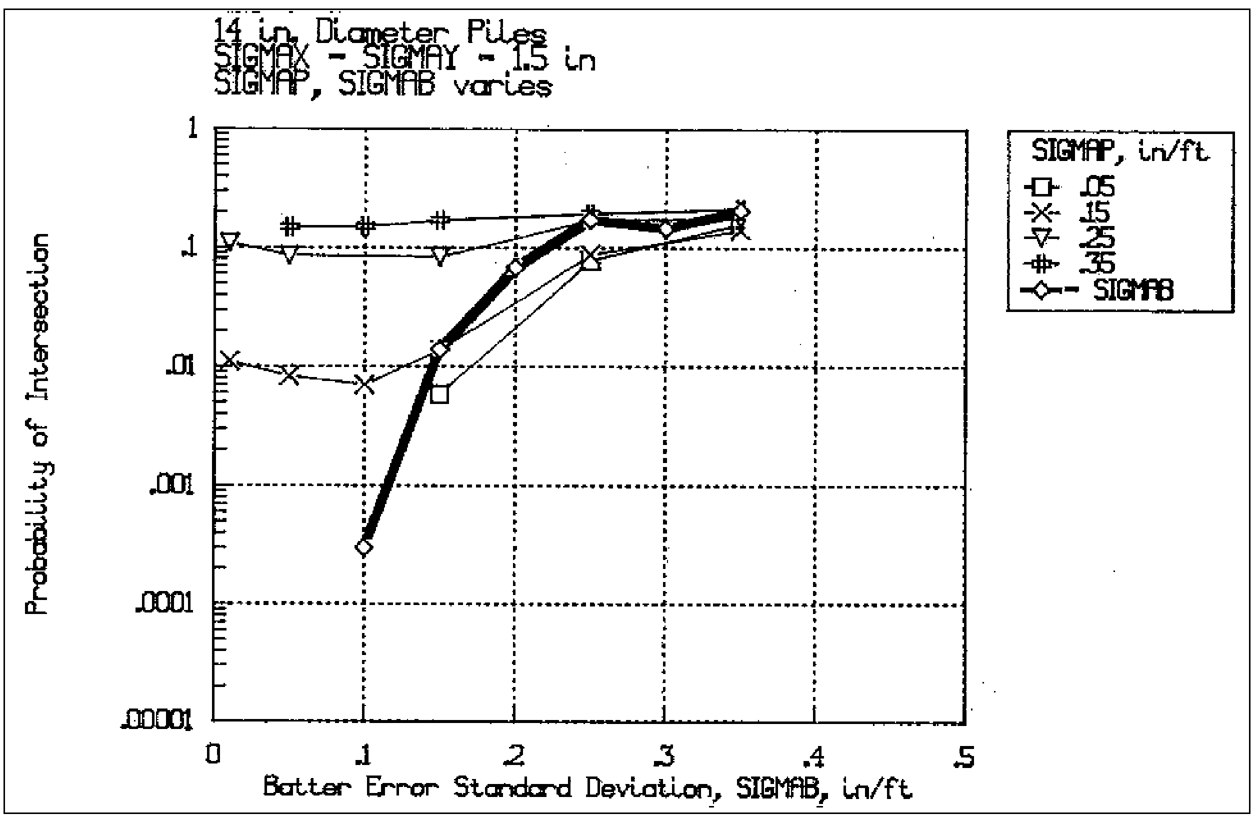


Figure 5-2. Effect of plumb and batter error on intersection probability

PILE LAYOUT TO MINIMIZE INTERFERENCE

EXAMPLE 3
PROBABILITY OF INTERSECTION FOR A GROUP OF OPPOSITE BATTER PILES WITH UNEQUAL SPACING

1. Introduction

This example illustrates how the software package CPGP would be used to obtain the probability distribution for the number of intersections for a group of 500 piles driven at opposite batters on a rectangular spacing. This is a case for which a chart solution is not available.

2. Given Conditions

Five hundred piles support a group of dam monoliths. There are fifty columns of ten piles each. Piles in each column are spaced 6 ft apart (AY = 4) and the columns are spaced 4 ft apart (AX = 6). Piles are battered at 1H to 3V. Piles in alternate columns are driven on opposite batters. The vertical projection of the pile length is 100 ft and the piles are 16 in. in diameter.

3. Probability of Intersection for a Single Interior Pile

The probability of intersection for a single interior pile is determined using the simulation module in CPGP. The input screen is shown in Figure 6-1. Default values are used for the number of trials, the maximum error, the standard deviations of the placement and alignment variables, and the camber parameter. The run-time screen at the end of the simulation is shown in Figure 6-2. The estimated probability of intersection for a single interior pile was found to be 0.001597, or approximately 0.0016. Note that replicating this example will give a somewhat different value as a different set of random numbers is generated for every run of the program. The true value for the probability of intersection is a random variable with expected value of 0.001597 and a standard deviation or *standard error* that decreases in proportion to the square root of the number of iterations. The simulation terminated when the standard error was less than 15 percent of the estimated probability, or in this case, 0.000242 (14.9 percent). Assuming the true probability to be normally distributed about the best estimate, the confidence limits on P[I] are obtained (Table 6-1). For practical purposes, there is a relatively high degree of confidence that the required probability lies between 0.001 and 0.002, which is sufficient information for decision-making, and the value of Pr[I] will be taken at 0.001597.

4. Output File

Detailed information regarding what occurred during the simulation can be obtained by inspection of the program output file shown in Figure 6-3. For each intersection, this file provides the trial number, the number of the pile struck by the interior pile (pile 5), the coordinates of the intersection, and the updated probability and error bounds. It is observed that all of the intersections occurred with piles 2 and 8, the piles ahead and behind the center pile in the y direction, which are more closely spaced than those in the x direction. The shallowest intersection took place at a depth of 68.8 ft, with most intersections occurring in the 80 to 95 ft range. The calculated probability of intersection varies from 0.001091 to 0.003333 early in the simulation; as the number of trials increases, the error bounds tighten and the probability of intersection fluctuates by progressively smaller increments.

5. Probability Distribution for the Group

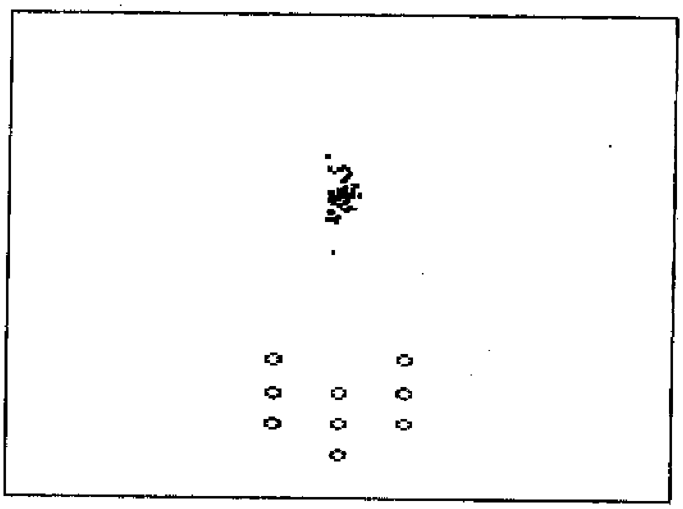
The probability distribution for the number of intersections on the group is determined using total intersection module in CPGP. The program screen is shown in Figure 6-4. The program is provided with the probability of intersection for a single pile (0.001597), the number of rows (10), and the number of columns (50). The program calculates the number of equivalent interior piles (441), the expected number of intersections in the entire

```
PILE INTERFERENCE SIMULATOR                        T. F. Wolff
Last revision -- March 27, 1990                    MICHIGAN STATE UNIVERSITY

Enter Output File Name
? b:out
                                   Use Up and Down Cursor to Select Data to Edit
                                   Use Left or Right Cursor to Provide Input

Number of trials                                 ntrials =     30000
Maximum Error                                   ErrorMax =      15.0   percent
Pile spacing in x direction,                          ax =      6.00   feet
Pile Spacing in y direction,                          ay =      4.00   feet
Pile diameter,                                         D =     16.00   inches
Vertical embedment,                                    L =    100.00   feet
Std dev of x error,                               sigmax =      1.50   inches
Std dev of y error,                               sigmay =      1.50   inches
Std dev of plumb error,                           sigmap =      0.15   inches per foot
Std dev of batter error ,                         sigmab =      0.10   inches per foot
Camber / sweep,                                   camber =     0.125   inches per 10 ft
Batter, 1 H to (+)B1 V  (0 for vert)                  B1 =       3.0
Batter of Adj. Row, 1H:B2V (- for opp)                B2 =      -3.0
                                                         *** GO ***
```

Figure 6-1. Input screen



```
                                                                  ax =   6
                                                                  ay =   4
                                                                   D =   16
                                                                   L =   100
                                                              sigmax =   1.5
                                                              sigmay =   1.5
                                                              sigmap =   .15
                                                              sigmab =   .1
                                                              camber =   .125
                                                             batter1 =   3
                                                             batter2 =  -3

28172 Trials       45 Intersections

          p - (1 std error) = 0.001359
PROBABILITY OF INTERSECTION = 0.001597             1 in      626
          p + (1 std error) = 0.001835
       normalized std error =  14.9 %
ANALYSIS COMPLETE -- RUN NEW CASE ? (Y or N) ? █
```

Figure 6-2. Run-time screen for PILINT

Table 6-1
Confidence Limits

Error Bounds	Value	Bounds	Percent Confidence
± 1σ	0.001597±0.000242	0.001359 0.001835	68.27 %
± 2σ	0.001597±0.000484	0.001113 0.002081	95.45 %
± 3σ	0.001597±0.000726	0.000871 0.002323	99.73 %

group (0.704), and the probability distribution on the number of intersections. From the program output, it is seen that there is approximately a 49 percent chance of no intersection, a 35 percent chance of one intersection, a 12 percent chance of two intersections, a 3 percent chance of three intersections, and a very small chance of four or more intersections. This is somewhat in excess of the suggested criteria of $E[I] < 0.5$; however, it may be acceptable, and would require an engineering judgment based on the risks and consequences of one or two intersections occurring.

```
PILE INTERFERENCE SIMULATOR          T. F. Wolff
Version: March 27, 1990              Michigan State University
Run at: 15:10:53 on 09-11-1990

x spacing      ax =  6  feet
y spacing      ay =  4  feet
Diameter        D =  16  inches
Length          L =  100  feet
x error    sigmax =  1.5  inches
y error    sigmay =  1.5  inches
plumb err  sigmap =  .15  in/ft
batter err sigmab =  .1  in/ft
camber     camber =  .125  in per 10 ft
batter         1H :  3 V
adj batter     1H : -3 V
```

Trial	Hit	Pile	x	y	z	p-	Prob	p+	%Error
300	1	8	0.3	26.8	77.2	0.000006	0.003333	0.006661	99.83
924	2	2	-1.1	28.1	87.6	0.000636	0.002165	0.003693	70.63
1311	3	2	0.6	28.1	87.2	0.000969	0.002288	0.003608	57.67
1797	4	2	-0.2	27.4	87.5	0.001114	0.002226	0.003338	49.94
3060	5	8	0.5	32.1	92.0	0.000904	0.001634	0.002364	44.68
5499	6	8	0.2	27.0	77.6	0.000646	0.001091	0.001536	40.80
5784	7	8	0.4	30.8	89.3	0.000753	0.001210	0.001667	37.77
5980	8	2	0.5	26.7	86.1	0.000865	0.001338	0.001810	35.33
7182	9	8	0.8	31.4	88.3	0.000836	0.001253	0.001671	33.31
7276	10	2	-1.3	25.8	83.0	0.000940	0.001374	0.001809	31.60
8960	11	2	1.6	28.8	90.4	0.000858	0.001228	0.001598	30.13
11337	12	8	0.4	29.6	88.0	0.000753	0.001058	0.001364	28.85
11428	13	2	0.4	28.8	89.2	0.000822	0.001138	0.001453	27.72
11441	14	2	1.0	30.0	93.5	0.000897	0.001224	0.001551	26.71
12001	15	2	-0.4	25.9	80.8	0.000927	0.001250	0.001572	25.80
12905	16	8	-1.1	26.6	77.2	0.000930	0.001240	0.001550	24.98
13911	17	8	0.1	30.4	86.5	0.000926	0.001222	0.001518	24.24
13919	18	8	-0.4	28.4	80.1	0.000989	0.001293	0.001598	23.55
14000	19	2	-1.0	28.2	87.4	0.001046	0.001357	0.001668	22.93
14318	20	8	-0.8	31.5	91.3	0.001085	0.001397	0.001709	22.35
15217	21	8	-0.1	32.0	93.0	0.001079	0.001380	0.001681	21.81
15918	22	2	0.3	27.7	85.4	0.001088	0.001382	0.001677	21.31
16448	23	8	-1.1	32.0	91.6	0.001107	0.001398	0.001690	20.84
16867	24	2	-0.9	25.7	82.5	0.001133	0.001423	0.001713	20.40
17227	25	2	-0.3	29.2	88.2	0.001161	0.001451	0.001741	19.99
17297	26	2	1.0	28.6	89.8	0.001209	0.001503	0.001798	19.60
17332	27	8	-0.7	28.2	83.7	0.001258	0.001558	0.001857	19.23
17917	28	2	-0.7	27.8	84.7	0.001268	0.001563	0.001858	18.88
18000	29	8	-0.6	25.4	73.2	0.001312	0.001611	0.001910	18.55
18904	30	2	1.0	29.3	91.6	0.001297	0.001587	0.001876	18.24
19280	31	8	0.4	30.7	88.2	0.001319	0.001608	0.001896	17.95
19954	32	8	-0.2	28.5	80.0	0.001320	0.001604	0.001887	17.66
20083	33	2	-0.8	21.4	68.8	0.001357	0.001643	0.001929	17.39
20575	34	8	0.2	29.4	85.5	0.001369	0.001652	0.001936	17.14
21203	35	8	0.6	30.9	88.4	0.001372	0.001651	0.001930	16.89
21278	36	8	-0.4	32.0	90.4	0.001410	0.001692	0.001974	16.65
22034	37	2	0.7	27.2	87.3	0.001403	0.001679	0.001955	16.43
22149	38	8	-1.5	33.4	93.3	0.001438	0.001716	0.001994	16.21
22662	39	8	0.1	32.2	94.2	0.001446	0.001721	0.001996	16.00
23256	40	8	1.1	27.0	74.1	0.001448	0.001720	0.001992	15.80
24535	41	2	-0.9	28.3	93.0	0.001410	0.001671	0.001932	15.60
27174	42	2	-1.1	28.9	91.8	0.001307	0.001546	0.001784	15.42
27571	43	8	0.6	31.9	89.4	0.001322	0.001560	0.001797	15.24
27869	44	2	1.3	29.9	93.8	0.001341	0.001579	0.001817	15.06
28172	45	8	0.3	29.2	83.3	0.001359	0.001597	0.001835	14.90

Figure 6-3. Listing of output file

```
                         Program PILTOTAL
          Total Probability of Intersection for a pile group
          T. F. Wolff - Michigan State University - April 1990

 Number of rows,        m =   ? 10       Number of Piles             =  500
 Number of columns,     n =   ? 50       Equivalent interior piles =  441
 Prob of Intersection                    Expected Number
   for one pile,        p =   ? .001597    of Intersections, E[I] =  .704277

 Pr(I= 0) = 0.494466    Pr(I> 0) = 0.505534    Pr(I<= 0) = 0.494466
 Pr(I= 1) = 0.348241    Pr(I> 1) = 0.157293    Pr(I<= 1) = 0.842707
 Pr(I= 2) = 0.122629    Pr(I> 2) = 0.034664    Pr(I<= 2) = 0.965336
 Pr(I= 3) = 0.028788    Pr(I> 3) = 0.005876    Pr(I<= 3) = 0.994124
 Pr(I= 4) = 0.005069    Pr(I> 4) = 0.000807    Pr(I<= 4) = 0.999193
 Pr(I= 5) = 0.000714    Pr(I> 5) = 0.000093    Pr(I<= 5) = 0.999907
 Pr(I= 6) = 0.000084    Pr(I> 6) = 0.000009    Pr(I<= 6) = 0.999991
 Pr(I= 7) = 0.000008    Pr(I> 7) = 0.000001    Pr(I<= 7) = 0.999999
 Pr(I= 8) = 0.000001    Pr(I> 8) = 0.000000    Pr(I<= 8) = 1.000000
 Pr(I= 9) = 0.000000    Pr(I> 9) = 0.000000    Pr(I<= 9) = 1.000000
 Pr(I=10) = 0.000000    Pr(I>10) = 0.000000    Pr(I<=10) = 1.000000

Type Q to Quit, or Press Return to Run Again?
```

Figure 6-4. Run-time screen for PILTOTAL